

Issues in Complex Structure Moduli Inflation

Hiroataka Hayashi¹, Ryo Matsuda² and Taizan Watari²

¹*Instituto de Física Teórica UAM/CSIC, Cantoblanco, 28049 Madrid, Spain*

³*Kavli Institute for the Physics and Mathematics of the Universe, University of Tokyo,
Kashiwa-no-ha 5-1-5, 277-8583, Japan*

Abstract

Supersymmetric compactification with moderately large radius ($\text{Re} \langle T \rangle \sim \mathcal{O}(10)$ or more) not only accommodates supersymmetric unification, but also provides candidates for an inflaton in the form of geometric moduli; the value of $\text{Re} \langle T \rangle > 1$ may be used as a parameter that brings corrections to the inflaton potential under control. Motivated by a bottom-up idea “right-handed sneutrino inflation” scenario, we study whether complex structure moduli can play some role during the slow-roll inflation and/or reheating process in this moderately large radius regime. Even when we allow a tuning introduced by Kallosh and Linde, the barrier of volume stabilization potential from gaugino condensation racetrack superpotential can hardly be as high as $(10^{16} \text{ GeV})^4$ for generic choice of parameters in this regime. It is also found that even very small deformation of complex structure during inflation/reheating distorts the volume stabilization potential, so that the volume stabilization imposes tight constraints on large-field inflation scenario involving evolution of complex structure moduli. A few ideas of satisfying those constraints in string theory are also discussed.

1 Introduction

Inflation provides an ideal opportunity to catch a glimpse of physics at very high energy. Time-evolution of an inflaton is sensitive to Planck-scale suppressed corrections, in small field inflation models and large field models alike; although the energy density during inflation was not as high as $\rho \sim M_{\text{Pl}}^4$, where $M_{\text{Pl}} \simeq 2.43 \times 10^{18} \text{GeV}$, we still need to have theoretical control over coherent deformation of the universe (inflaton field value) that is more than $\mathcal{O}(1)$ in M_{Pl} unit. String theory being virtually the only known candidate for calculable quantum theory of gravity, it is worthwhile to study what the “control” is like, and what the microscopic picture of the “coherent deformation of the universe” is like, in string theory.

String theory has another advantage of being an “all-in-one package”; one cannot choose a set of compactification manifold and brane configuration for inflation and another for physics of quarks and leptons. This is in sharp contrast against the traditional framework of low-energy effective field theory. Models for inflaton and those for our particles (quarks, leptons, photons etc.) can be discussed separately in low-energy effective field theory, and a set-up necessary for successful inflation rarely imposes constraints on the models of elementary particles, or vice versa. In string theory, however, we need to have both the inflaton and the particle sector together in a common package of compactification manifold and brane configuration, and the post inflationary evolution of the universe¹ needs to be worked out in the common package. This is highly a non-trivial study, but such a tight constraint from string theory can be exploited to uncover the history of the universe in the early stage.

In this article, we are motivated by a bottom-up idea that the inflaton may be identified with the supersymmetry partner of right-handed neutrinos (right-handed sneutrino inflation scenario) [1]. Post-inflationary thermal history has been studied very well, and it is known that this scenario passes various constraints from phenomenology. This scenario comes with some assumptions on Kähler potential, however, and it is a legitimate question whether this bottom-up idea can be accommodated in some string compactification or not. Right-handed neutrino chiral multiplets are known to be identified with a part of complex structure moduli in F-theory compactifications in the matter parity scenario [2],² and this observation motivates us to study what it takes for complex structure moduli to drive (or at least to be relevant to) inflation in Type IIB/F-theory compactifications in general.

¹Especially it should have a graceful exit from inflation and subsequent reheating to our particle sector, but not too much to other sectors (incl. gravitino, axino, (s)axion, hidden sector particles and moduli fields).

²This is equivalent, through string duality, to a statement that vector bundle moduli are identified with right-handed neutrino chiral multiplets in Heterotic string compactifications [3].

Mirror symmetry indicates that the Kähler potential of some kind of complex structure moduli has properties that we expect for that of the Kähler moduli, as pointed out also recently in the context of inflation [4] (see also [5, 6]).³ Since the Kähler potential of Kähler moduli has an approximate shift symmetry in the regime supergravity approximation is valid, the Kähler potential of complex structure moduli should also have an approximate shift symmetry near the large complex structure limit. When flux is introduced in Type IIB/F-theory compactifications, complex structure moduli dependent superpotential is also generated [8]. The Majorana mass of right-handed neutrinos, which is tied with the mass of the complex structure moduli, sets a scale somewhat below the Kaluza–Klein scale [9, 2], which is a necessary ingredient of inflation [1]. A natural question is, then, whether this set-up can be relevant, in any way, to the slow-roll inflation process. We address this question in this article, and intend to share our thoughts with experts around the world.

In section 2, we combine the discussion of Refs. [10] and [11] and arrive at two observations; i) the stabilization of Kähler moduli during inflation sets a stringent constraint on the evolution of complex structure moduli during inflation, and ii) mass-eigenstates at the vacuum reached after inflation are generically mixtures of Kähler moduli and complex structure moduli fields. In order to work out the constraint in the observation (i) in detail, one needs to estimate the prefactor in the gaugino condensation superpotential, first of all; therefore we refine the existing estimate of the prefactor in section 3. In section 4, we derive constraints on how much the field value of complex structure moduli during inflation/reheating can be different from the vacuum value, provided the Kähler moduli field is stabilized by the racetrack superpotential from two gaugino condensates; we employed a phenomenological approach so that the constraint can be stated without referring to the choice of a Calabi–Yau 3-fold for Type IIB compactification. In sections 5.1 and 5.2, such a phenomenological and robust constraint is translated into the language of Type IIB flux compactification superpotential, and studied in combination with how to realize the tuning of Kallosh–Linde for supersymmetric volume stabilization [10]. We see that the constraint is so stringent that very little e-fold can be earned generically by evolution of the complex structure moduli fields. In section 5.3, we discuss a few possibilities of making the right-handed sneutrino inflation scenario and/or complex structure moduli driven inflation still at work, while satisfying the constraint.

³Recently, there have been also progress in large field inflation scenarios using other moduli in string compactification [7].

2 Interplay between Kähler and Complex Structure Moduli under the Kallosh–Linde Tuning

2.1 Volume Stabilization and Large Field/High Scale Inflation

One of the most important lessons from studies on D-brane inflation almost a decade ago was that stabilization of the Kähler moduli of a compactification is essential [12, 10] (see also [13]). This remains true in the current situation.

To get started, let us remind ourselves that the scalar potential for the complex structure moduli in flux compactification was shown to be positive definite in [11] under two assumptions. The first assumption is that the superpotential of the 4D effective theory is independent of Kähler moduli, $\partial_T W = 0$, and the other is that the Kähler potential of the Kähler moduli T is of no-scale type:

$$\frac{K}{M_{\text{Pl}}^2} = -3 \ln[\text{Re}(T)] + \mathcal{K}^{(\text{cpx})}(z, z^\dagger). \quad (1)$$

Then the superpotential depends only on complex structure moduli z , $W = W^{(\text{cpx})}(z)$ (in the context of Type IIB Calabi–Yau orientifolds, z consists of both the axion-dilaton chiral multiplet τ and complex structure moduli ζ of a 3-fold).⁴ The positive-definiteness of the potential is simply due to cancellation within the second term of

$$V = e^{K/M_{\text{Pl}}^2} K^{\bar{z}z} |D_z W|^2 + e^{K/M_{\text{Pl}}^2} \left[K^{\bar{T}T} |D_T W|^2 - 3 \left| \frac{W}{M_{\text{Pl}}} \right|^2 \right], \quad (2)$$

where

$$K^{\bar{T}T} M_{\text{Pl}}^2 = \frac{(T + \bar{T})^2}{3}, \quad \frac{K_T}{M_{\text{Pl}}^2} = -\frac{3}{(T + \bar{T})}; \quad (3)$$

the scalar potential V is given by the positive definite first term after the cancellation.

The Kähler moduli needs to be stabilized, however. Since the T -independence of the superpotential is a crucial ingredient of the assumptions leading to the positive-definiteness, one should not expect this positive-definiteness to hold true, even after non-perturbative effects generate T -dependent terms in the effective superpotential. Suppose for now⁵ that

⁴To readers unfamiliar with string theory: it will be helpful to read a brief review in section 5.1.1, in order to get the feeling of what we have in mind for $\mathcal{K}^{(\text{cpx})}(z, z^\dagger)$ and $W^{(\text{cpx})}(z)$.

⁵ It looks as if we assume that there is only one Kähler modulus here, while it is well-known that the orientifold-even part of $h^{1,1}$ of a Calabi–Yau 3-fold for Type IIB compactification ($h^{1,1}$ of Calabi–Yau 4-fold for F-theory) is not always 1. Even in cases with $h_+^{1,1} > 1$ ($h^{1,1} > 1$ resp.), however, it is possible that

the Kähler potential remains to be the one in (1), and the effective superpotential is of the form

$$W^{(\text{tot})} = W^{(T)}(T, z) + W^{(\text{cpx})}(z). \quad (4)$$

There remains partial cancellation in the second term of (2), but the cancellation is no longer complete:

$$V = e^{K/M_{\text{Pl}}^2} \left[K^{\bar{z}z} |D_z W|^2 + K^{\bar{T}T} |\partial_T W|^2 + \frac{K^{\bar{T}T}}{M_{\text{Pl}}^2} \{ (\partial_T W) K_{\bar{T}} \bar{W} + \text{h.c.} \} \right]. \quad (5)$$

The last term containing $\{ (\partial_T W) K_{\bar{T}} \bar{W} + \text{h.c.} \}$ can be either positive or negative, and the scalar potential V is not guaranteed to be positive definite.

When the coherent value of the effective superpotential W during inflation is too much different from the vacuum value, the potential for Kähler moduli T stabilization is deformed so much that the internal space may start to decompactify. The condition of volume stabilization therefore has to be imposed on any kinds of complex structure moduli inflation models (particularly large field/high scale inflation models). We will elaborate more on this issue in later sections.

In order to support inflation in the complex structure moduli field with a high energy scale involved, it is better to stabilize the volume at as high energy scale as possible; the original motivation of the right-handed sneutrino inflation scenario [1] was to implement chaotic inflation indeed. To stabilize the Kähler moduli at supersymmetric level, we need a fine tuning [10], and indeed only one fine tuning is necessary. This is to assume that

$$\langle W^{(T)}(\langle T \rangle, \langle z \rangle) + W^{(\text{cpx})}(\langle z \rangle) \rangle = 0 \quad (6)$$

at the minimum (well after the last inflation), where this condition is imposed for the value of $z = \langle z \rangle$ and $T = \langle T \rangle$ determined by

$$\langle \partial_T W^{(T)}(T, z) \rangle = 0, \quad \langle \partial_z (W^{(T)}(T, z) + W^{(\text{cpx})}(z)) \rangle = 0. \quad (7)$$

With these conditions, $z = \langle z \rangle$ and $T = \langle T \rangle$ give a minimum of V with vanishing cosmological constant (in the tree-level approximation of 4D supergravity).⁶ We refer to the condition (6) as Kallosh-Linde tuning in this article.

there is only one independent Kähler modulus field after flux is introduced, because the Fayet-Iliopoulos parameter ξ of a U(1) gauge field on a 7-brane is of the form $\xi \sim \int \omega \wedge F$ for a non-trivial flux F on the 7-brane world-volume [14]. It is still an assumption in such cases that the effective Kähler potential of such an independent Kähler modulus remains to be in the form of (1).

⁶Supersymmetry breaking and non-vanishing cosmological constant are ignored as subleading effects throughout this article.

For small fluctuations around this supersymmetric minimum, where the vacuum energy also vanishes, the scalar potential is positive definite. When the field value differs too much from their vacuum value, however, the scalar potential is no longer guaranteed to be positive, as we have already discussed below (5). The question is how much deformation in the moduli fields are allowed, and this is what we study in the following sections.

2.2 Comments on Reheating Process

One can also work out the mass matrix around such a minimum,⁷ which is relevant information on the reheating process. The quadratic part of the action at the minimum is in the form of

$$\mathcal{L}^{(2)} = -\langle K_{T\bar{T}} \rangle |\partial(\delta T)|^2 - \langle K_{z\bar{z}} \rangle |\partial(\delta z)|^2 - ((\delta T)^\dagger, (\delta z)^\dagger) \cdot M \cdot \begin{pmatrix} \delta T \\ \delta z \end{pmatrix}, \quad (8)$$

where $\delta T = T - \langle T \rangle$, $\delta z = z - \langle z \rangle$ and the mass matrix is given by

$$M = e^{K/M_{\text{Pl}}^2} K^{\bar{T}T} \begin{pmatrix} \frac{|\partial_T^2 W|^2}{(\partial_z \partial_T W)(\partial_T^2 W)} & \frac{\overline{(\partial_T^2 W)}(\partial_z \partial_T W)}{|\partial_z \partial_T W|^2} \\ +e^{K/M_{\text{Pl}}^2} K^{\bar{z}z} \begin{pmatrix} \frac{|\partial_T \partial_z W|^2}{(\partial_z^2 W)(\partial_T \partial_z W)} & \frac{\overline{(\partial_T \partial_z W)}(\partial_z^2 W)}{|\partial_z^2 W|^2} \end{pmatrix} \end{pmatrix} \quad (9)$$

evaluated at the minimum. The physical mass matrix m^2 is obtained by rescaling the fluctuations δT and δz so that the kinetic terms become canonical at the minimum (i.e., by sandwiching M by $\text{diag}(\langle K^{\bar{T}T} \rangle, \langle K^{\bar{z}z} \rangle)^{1/2}$). Since it is common place in string compactifications⁸ that the term $W^{(T)}(T, z)$ depends on the moduli z , there are mixing terms in the mass matrix. All the mass eigenstates around the minimum therefore possess interactions of both δT and δz , when all the relevant mass scales are high (as assumed in the context of tuning (6)). If the fluctuation δz has a renormalizable coupling with particles in the Standard Model (remember that the right-handed neutrinos do have one), this means that all the energy of coherent oscillation in δT and δz is converted quickly into that of radiation in the Standard

⁷ Note that the following discussion as well as that in the next section remains valid even when the Kähler potential is not precisely the same as the no-scale type $\propto -3 \ln[\text{Te}(T)]$, or there is more than one independent Kähler moduli chiral multiplet. No matter how many independent Kähler moduli are left after taking account of the D-term from fluxes on D-branes (see footnote 5), their masses can be much larger than the gravitino mass as long as the tuning (6) is achieved somehow [15].

⁸Footnote 11 provides a pedagogical explanation for this.

Model, and the cosmological moduli problem [16] is avoided;⁹ matter parity is ignored for simplicity of the argument for now, and we will come to this point shortly.

When the physical masses $m_{T\bar{T}}^2$ and $m_{z\bar{z}}^2$ are comparable, the δT – δz mixing due to the z -dependence of $W^{(T)}(T, z)$ can be sizable, as this will become clearer after the discussion in section 3. It is often perceived that the interactions between Kähler moduli and complex structure moduli are suppressed by Planck scale (or string scale M_s); that is certainly true, but in the cases where the relevant energy scale of gaugino condensation (Kähler moduli stabilization) is very high (as required in high scale inflation), mass mixing $W \supset m (\delta T) \cdot (\delta z)$ comes at the order of $m \approx (\text{high scale})^2/M_s$, and the renormalizable interactions such as $W \supset \lambda(\delta T)(\delta z)^2$ have coefficients $\lambda \approx (\text{high scale})/M_s$. Therefore, decay processes through this mixing will have rates of order $\Gamma \approx (\text{high scale})^3/M_s^2$, which is often sizable in the context of thermal history of the universe after inflation.

In a more realistic set-up of supersymmetric compactification, one might wish to introduce matter parity for proton stability. Such a scenario assumes that a \mathbb{Z}_2 symmetry is restored at the minimum $z = \langle z \rangle$ and $T = \langle T \rangle$, so that all the fluctuations are classified into the \mathbb{Z}_2 -even and \mathbb{Z}_2 -odd sectors; let $\hat{\Phi}_+$ and $\hat{\Phi}_-$ denote such even and odd mass eigenstates, respectively. The mass/kinetic mixing as above is found only within these two sectors separately. Because there can be interactions such as $W \supset \hat{\Phi}_+ \hat{\Phi}_- \hat{\Phi}_-$, various chains of preheating and (on-shell or virtual) cascade decay processes may be at work, which means that energy can be transferred from the \mathbb{Z}_2 -even sector to the odd sector, or vice versa. Thus realistic story of reheating process would not be as simple as in the discussion above without matter parity. How much fraction of coherent oscillation energy is in the \mathbb{Z}_2 -even sector of $(\delta T, \delta z)$ at the end of inflation depends on details of inflation models. The preheating and decay processes depend very much of the spectrum of the moduli fields at the vacuum. It also makes a big difference in the thermal history whether there are \mathbb{Z}_2 -even moduli fields with renormalizable couplings with the Standard Model particles ($W \supset \hat{\Phi}_+ \cdot H_u \cdot H_d$ or $\supset \hat{\Phi}_+ \cdot \mathbf{5} \cdot \mathbf{\bar{5}}$ (vector-like SUSY-breaking messengers)). For all these model-dependence, we do not try to develop discussion on the reheating process further than this in this article.

⁹The cosmological moduli problem is avoided also by the large moduli mass (see footnote 7).

3 Setting Scales

In flux compactification of Type IIB string/F-theory, it is known that the Gukov–Vafa–Witten superpotential [8],

$$W^{(\text{cpx})}(z) = W_{GVW} = c \int_X G \wedge \Omega_X, \quad (10)$$

describes the effective superpotential of the complex structure moduli z (including dilaton) for some coefficient c . Non-perturbative effects may generate effective superpotential that depends on the Kähler moduli T ; each one of such terms is of the form

$$W^{(T)}(T, z) = W_{np} = A(z)e^{-dT}, \quad (11)$$

and $d = 2\pi/N$ in the case of gaugino condensation of 4D $SU(N)$ super Yang–Mills theory; the prefactor A may depend on the complex structure moduli field z in principle, and at least it does on the choice of normalization of Ω_X . Supersymmetric stabilization of the Kähler moduli is possible, if there are more than one term of the form (11) (the racetrack scenario), and the tuning (6) is assumed. In order to work out the decompactification constraint on inflation discussed in the previous section, we need to know the coefficients c and A .

The coefficient c has been determined in [17] already. The procedure is to carry out dimensional reduction from 10D to 4D, and work out the 4D scalar potential of complex structure moduli in flux compactification first. The value of c is determined so that the 4D effective supergravity potential reproduces the potential obtained through the reduction. The result is that [17]

$$c = M_{\text{Pl}}^3 \frac{1}{\sqrt{4\pi}} \quad (12)$$

for Type IIB orientifold on a Calabi-Yau 3-fold X , when one uses the Kähler potential

$$\frac{K}{M_{\text{Pl}}^2} = -\ln \left[i \int_X \Omega \wedge \bar{\Omega} \right] - \ln [(\tau - \bar{\tau})/i] - 2 \ln \left[\frac{1}{3!g_s^{3/2}} \int_X \omega^3 \right]. \quad (13)$$

Here, $\tau := C^{(0)} + ie^{-\phi} = C^{(0)} + ig_s^{-1}e^{-\tilde{\phi}}$ is the axion-dilaton chiral multiplet, g_s the vacuum value of e^ϕ , and ω the Kähler form¹⁰ on X . The last term is equivalent to the first term $-3 \ln(\text{Re}(T))$ in (1), and the first two terms are identified with $\mathcal{K}^{(\text{cpx})}$.

¹⁰The Kähler form ω refers to the one in the Einstein frame metric g_E of Type IIB 10-dimensional supergravity; we assumed that the Weyl rescaling from the string frame metric is given by $g_S \rightarrow e^{(\phi - \langle \phi \rangle)/2} g_E$. We understand that ω has been made dimensionless by using the dimensionless coordinates \underline{y}^m .

We have adopted a convention to make everything dimensionless in $\int_X G \wedge \Omega$, by rendering the space coordinates y^m dimensionless; $y^m \rightarrow \underline{y}^m = y^m/\ell_s$, $\ell_s := 2\pi\sqrt{\alpha'} =: 1/M_s$. Three-form fluxes of Type IIB string theory are quantized as

$$\frac{1}{\ell_s^2} \int F^{(3)} \in \mathbb{Z}, \quad \frac{1}{\ell_s^2} \int H^{(3)} \in \mathbb{Z}, \quad (14)$$

and are therefore turned into dimensionless integers $\int F^{(3)} = n^R \in \mathbb{Z}$ and $\int H^{(3)} = n^{NS} \in \mathbb{Z}$ in the dimensionless coordinate setting. Thus,

$$\int_X G \wedge \Omega = \sum_a (n_a^R - \tau n_a^{NS}) \Pi_a, \quad (15)$$

with integer flux quanta $n_a^R, n_a^{NS} \in \mathbb{Z}$ and dimensionless period integrals Π_a 's ($a = 1, \dots, 2(h_+^{2,1} + 1)$).

The prefactor A in (11) for gaugino condensation is estimated by matching the moduli scalar potential from gaugino condensation in 4-dimensions to a *supergravity* potential of the effective theory of moduli fields. We work on the case of gaugino condensation of $SU(N)$ super Yang–Mills theory originating from an $SU(N)$ gauge group on a stack of N D7-branes.

This is not the first time that this problem is addressed; Ref. [18] did that in Heterotic string compactifications, for example. It will be easy to see that this problem involves subtlety, because there are non-decoupling effects in the $M_{\text{Pl}} \rightarrow \infty$ limit. Consider a factor e^{K/M_{Pl}^2} , for example. It is just 1, if we simply set $M_{\text{Pl}} \rightarrow \infty$, but if we are to rewrite K/M_{Pl}^2 by using (13), it remains a factor with non-trivial dependence on various moduli fields. When we use the results of gaugino condensation in the rigid supersymmetry limit, we need to be careful.

To properly appreciate the subtleties in the context of Type IIB compactifications, let us start off by reminding ourselves of the well-known procedure of dimensional reduction. The Einstein–Hilbert term of the Type IIB 10D Einstein frame action is given by

$$S_E \supset \frac{2\pi}{\ell_s^2 g_s^2} \int d^4x \int_X d^6 \underline{y} \sqrt{-g_E} R_E, \quad (16)$$

where the six coordinates on X have been made dimensionless, and the Einstein frame metric g_E is used. Dimensional reduction leads to

$$S_{E|4} \supset \frac{2\pi}{\ell_s^2 g_s^2} \int d^4x \sqrt{-g_{E|4}} \omega^3 R_{E|4} + \dots, \quad (17)$$

where $g_{E|4}$ is the restriction of the 10D Einstein frame metric g_E . We call this action on 4-dimensions as that of *reduction frame*.

This reduction frame action can be cast into an Einstein frame action by Weyl-rescaling. We adopt the following rescaling,

$$g_{E|4} \longrightarrow g_4 \times \frac{\langle \omega \rangle^3}{\omega^3}, \quad (18)$$

so that all of the metrics g_4 , $g_{E|4}$, g_E and g_S have the same normalization in their vacuum expectation values. With this Weyl rescaling, we obtain

$$S_4 \supset \frac{4\pi \langle \omega \rangle^3}{2\ell_s^2 g_s^2} \int d^4x \sqrt{-g_4} R_4, \quad M_{\text{Pl}}^2 = \frac{4\pi \langle \omega \rangle^3}{\ell_s^2 g_s^2}. \quad (19)$$

It should be reminded that the reduction frame above is not the same as the frame that is used in writing down a 4D supergravity action on a superspace. Let a supergravity action on superspace be

$$S_{SS4} = \int d^4x \int d^2\Theta 2\mathcal{E} \left[\frac{3M_{\text{Pl}}^2}{8} (\overline{\mathcal{D}}^2 - 8R) e^{-\frac{K'+\Gamma}{3M_{\text{Pl}}^2}} + W + H 2 \text{tr}_N [W^\alpha W_\alpha] \right] + \text{h.c.}, \quad (20)$$

where W is the superpotential, K' the Kähler potential and H the gauge kinetic function which may depend on chiral multiplets holomorphically; for all other notations, see [19]. The Weyl rescaling from this superspace frame to the Einstein frame is

$$g_{SS4} \longrightarrow g_4 \times e^{\frac{K'}{3M_{\text{Pl}}^2}}. \quad (21)$$

If we are to use the Kähler potential K in (13) as K' in the superspace, the Weyl rescaling factor in (21) depends on the complex structure moduli fields, whereas the one in (18) does not. Even when it comes to the dependence on the Kähler moduli fields, the rescaling factor in (21) is proportional to

$$e^{\frac{K}{3M_{\text{Pl}}^2}} \propto e^{-\frac{2}{3} \ln \left[\frac{\omega^3}{g_s^{3/2}} \right]} = \frac{g_s}{\omega^2}, \quad (22)$$

which is clearly different from the rescaling factor in (18). Note also that the vacuum values of the metric g_{SS4} and $g_{E|4}$ in the two frames are not the same either, because $\langle e^{K'/3M_{\text{Pl}}^2} \rangle \neq 1$, unless we choose $K' = K - \text{const.}$, so that $\langle K' \rangle = 0$.

Reference [20] discussed the matching of gaugino condensation between [4D Einstein / superspace frame action with a vector multiplet] and [4D effective theory of moduli fields (in

the Einstein frame)], but the IIB-reduction frame was not used. We will thus translate the reduction frame action to the Einstein frame, and get the matching done, in the following.

As is well-known, the reduction of DBI action of a stack of N D7-branes gives rise to

$$S_{E|4} \supset \int d^4x d^2\theta \frac{T}{16\pi} 2\text{tr}_N[W^\alpha W_\alpha] + \text{h.c.}, \quad \text{Re}(T) = \frac{\omega^2}{g_s} \quad (23)$$

in the IIB-reduction frame; the Weyl rescaling (18) keeps the gauge field as it is, but the gaugino also needs to be rescaled by

$$\lambda_{E|4} \longrightarrow \lambda_{4,h} \times \left(\frac{\omega^3}{\langle \omega \rangle^3} \right)^{\frac{3}{4}} \quad (24)$$

so that the kinetic term is of the form

$$S_4 \supset \int d^4x \sqrt{-g_4} \left[-\frac{1}{4g_{YM}^2} 2\text{tr}[F_{\mu\nu} F_{\rho\sigma}] g_4^{\mu\rho} g_4^{\nu\sigma} - \frac{i}{g_{YM}^2} 2\text{tr}[\lambda_{4,h} \sigma^a e_a^\mu D_\mu \bar{\lambda}_{4,h}] \right] \quad (25)$$

in the Einstein frame; the coefficient of $2\text{tr}_N[W^\alpha W_\alpha]$ in S_4 is shifted to become

$$\frac{T}{16\pi} \longrightarrow \frac{1}{16\pi} \left[T + \frac{2N}{2\pi} \frac{3}{4} \ln \left[\frac{\omega^3}{\langle \omega \rangle^3} \right] \right] \quad (26)$$

because of the rescaling anomaly; $2N$ is the number of gaugino zero modes in a 1-instanton background. The relation

$$\langle \text{Re}(T) \rangle = \frac{4\pi}{g_{YM}^2} \quad (27)$$

remains the same, however.

The well-known result of gaugino condensation in rigid supersymmetry,

$$\frac{\langle 2\text{tr}_N[\lambda\lambda]_h \rangle}{32\pi^2} = N\Lambda_h^3, \quad (28)$$

should be understood as follows. First, gaugino $\lambda_{4,h}$ in the Einstein frame action with holomorphic normalization is used on the left-hand side, since it is the Einstein frame metric g_4 whose vacuum value becomes $\eta_{\mu\nu}$ (rather than g_{SS4} in the superspace frame), and that such terms as $[K_T(\partial_\mu T)]$ in the covariant derivative of gaugino $D_\mu \lambda$ do not play a role in determining the instanton zero mode configuration [18, 20].

Secondly, the dynamical scale Λ_h is given by $\Lambda_h^{3N} = \mu^{3N} e^{-\frac{8\pi^2}{g_{YM}^2(\mu)} + i\theta}$ using the gauge coupling $g_{YM}^2(\mu)$ in the Einstein frame action renormalized at scale μ . In terms of string compactification,

$$\Lambda_h^3 \simeq M_{KK}^3 e^{-\frac{2\pi}{N}T}. \quad (29)$$

Some power of $[\text{Re}(T)/\text{Re}\langle T \rangle]$ may be included on the right-hand side, but it is not important for the purpose of determining the prefactor A through matching. The renormalization scale μ was replaced by the Kaluza–Klein scale M_{KK} , because the D7-brane DBI action on a flat spacetime background is a good approximation only at energy scale above M_{KK} [18].¹¹

The Einstein frame 4D supergravity action in a theory with an $\text{SU}(N)$ vector multiplet has such terms as [19]

$$\begin{aligned} \sqrt{-g_4}^{-1} \mathcal{L}_4 \supset & e^{K/M_{\text{Pl}}^2} \left[3 \left| \frac{W}{M_{\text{Pl}}} \right|^2 \right. \\ & \left. - K^{\bar{j}i} \left[D_i W - H_{,i} e^{-\frac{K}{2M_{\text{Pl}}^2}} 2 \text{tr}[\lambda\lambda]_{4,h} \right] \left[\overline{D_j W} - \overline{H_{,j}} e^{-\frac{K}{2M_{\text{Pl}}^2}} 2 \text{tr}[\bar{\lambda}\bar{\lambda}]_{4,h} \right] \right], \end{aligned} \quad (30)$$

and the gauge kinetic function H is given by $T/(16\pi)$ in our context; the gauge kinetic function remains the same under the Weyl rescaling (21) between the superspace frame and Einstein frame. The idea of [20] for the matching is to replace the gaugino composite operators in (30) by the moduli dependent expectation value (29), and come up with an effective superpotential $W_{\text{eff.}}(T)$ so that the auxiliary F-term of the Kähler moduli chiral multiplet T reproduces the potential generated from (30).

We find that

$$W_{\text{eff.}} \sim W + M_{KK}^3 N^2 e^{-\frac{\langle K \rangle}{2M_{\text{Pl}}^2}} e^{-\frac{2\pi}{N}T}. \quad (31)$$

To see this, it would not be difficult to see that a simple computation

$$-(\partial_T H) 2 \text{tr}_N[\lambda\lambda]_{4,h} e^{-\frac{K}{2M_{\text{Pl}}^2}} = -2\pi N M_{KK}^2 e^{-\frac{2\pi}{N}T} e^{-\frac{K}{2M_{\text{Pl}}^2}} = \partial_T \left[N^2 M_{KK}^3 e^{-\frac{2\pi}{N}T} \right] e^{-\frac{K}{2M_{\text{Pl}}^2}} \quad (32)$$

is behind the identification of $W_{\text{eff.}}$, first of all. Secondly, we should remember that this matching is ultimately based on the the result of rigid supersymmetry (28), and we can determine how Λ_h^3 depends on T and $\langle T \rangle$ separately only through guess work. We chose to deal with M_{KK} and $e^{-\langle K \rangle/2M_{\text{Pl}}^2}$ in (31) as their vacuum values at the end, not as field-dependent functions, because we know that the effective superpotential of gaugino condensation depends on the Kähler moduli *field* T only in the form of $e^{-\frac{2\pi}{N}T}$.

¹¹ When 1-loop threshold corrections are included in the relation (27), the factor M_{KK}^3 is replaced by a more precise expression in the form of $[1/\ell_s \sqrt{\langle \omega \rangle}]^3$ times some dimensionless “value” that is generically of order unity. Since spectrum around the Kaluza-Klein scale depends on complex structure moduli z , so does the 1-loop threshold correction. The “value” should therefore actually be a function of z ; this is how the z -dependence originates in the gaugino condensation superpotential $W^{(T)}(T, z)$.

Note that the new term from gaugino condensation in the effective theory superpotential $W_{\text{eff.}}$ in (31) also contributes to the $-3|W_{\text{eff.}}/M_{\text{Pl}}|^2$ term in the effective theory scalar potential. This contribution in the effective theory cancels against the $K^{\bar{T}T}|K_T W_{\text{eff.}}|^2$ term in the effective theory scalar potential, which we kept out of (32), provided the Kähler potential of T is in the no-scale type. Cancellation also takes place on the other side of the matching: there is no such term as

$$\frac{|H|^2}{M_{\text{Pl}}^2} e^{-K/3M_{\text{Pl}}^2} \text{tr}[\lambda\lambda]_{4,h} \text{tr}[\bar{\lambda}\bar{\lambda}]_{4,h} \quad (33)$$

in the Einstein frame action $\sqrt{-g_4}^{-1} \mathcal{L}_4$ of supergravity with an $\text{SU}(N)$ vector multiplet [19]. Thus, there is no contradiction in the current set-up, when we adopt the matching condition (31).

The effective superpotential of the gaugino condensation is rewritten into a form that fits better for practical analysis. Noting that

$$M_{KK} \sim \frac{1}{\ell_s \sqrt{\langle \omega \rangle}} \sim M_{\text{Pl}} \frac{1}{\sqrt{4\pi}} \frac{g_s}{\langle \omega \rangle^2} = M_{\text{Pl}} \frac{1}{\sqrt{4\pi \text{Re} \langle T \rangle}}, \quad (34)$$

and that

$$e^{-\frac{\langle K \rangle}{2M_{\text{Pl}}^2}} \sim \left[\int \Omega \sqrt{2/g_s} \right] \times [\text{Re} \langle T \rangle]^{3/2}, \quad (35)$$

we rewrite the gaugino condensation contribution to $W_{\text{eff.}}$ as

$$W_{np}(T, z) = M_{\text{Pl}}^3 \frac{a N^2}{[4\pi \text{Re} \langle T \rangle]^{3/2}} e^{-\frac{2\pi}{N} T}, \quad (36)$$

with

$$a \sim \left[\int \Omega \times \sqrt{2/g_s} \right]. \quad (37)$$

The 1-loop threshold correction to the relation (27), which generically depends on the complex structure moduli fields z , is implemented in this factor a .

We now therefore conclude that the superpotential of the moduli effective theory is of the form

$$W_{\text{eff}} = M_{\text{Pl}}^3 \left[\sum_i \frac{1}{\sqrt{(4\pi \langle \text{Re}(T) \rangle)^3}} a_i N_i^2 e^{-\frac{2\pi}{N_i} T} + \frac{1}{\sqrt{4\pi}} \sum_a (n_a^R - \tau n_a^{NS}) \Pi_a \right], \quad (38)$$

where a_i 's are dimensionless and will remain of order unity, though it may depend on the complex structure moduli fields z 's, and on normalization of Ω_X .

As a sanity check, we use the effective superpotential (10, 12) and the effective Kähler potential (13) to estimate physical masses of the complex structure moduli, and see whether

the result is sensible. Ignoring the complex structure moduli dependence of the coefficient A in (11), the physical mass of complex structure moduli can be estimated by using $m_{z\bar{z}}^2 \sim e^{K/M_{\text{Pl}}^2} |K^{\bar{z}z} \partial_{\bar{z}}^2 W|^2$ [cf. (9)]:

$$\begin{aligned} m_{z\bar{z}}^2 &\sim \left[\frac{1}{i \int_X \Omega_X \wedge \bar{\Omega}_X} \frac{g_s}{2} \frac{g_s^3}{\langle \omega \rangle^6} \right] \left[\frac{\int_X \chi_a \wedge \bar{\chi}_b}{\int_X \Omega \wedge \bar{\Omega}} \right]^{-2} \frac{1}{4\pi} \left| \sum_a (n_a^R - \tau n_a^{NS}) \partial_z^2 \Pi_a \right|^2 M_{\text{Pl}}^2, \\ &\sim \frac{M_{\text{Pl}}^2}{4\pi} \frac{g_s^3}{\langle \omega \rangle^6} \sim \frac{g_s}{\ell_s^2 \langle \omega \rangle^3} \sim \left[\frac{g_s^{1/2} M_{KK}^3}{(1/\ell_s)^2} \right]^2, \end{aligned} \quad (39)$$

where $\{\chi_a\}$'s is a basis of (2,1) forms on a Calabi-Yau 3-fold X . When passing to the second line, we evaluated $(n_a^R - \tau n_a^{NS})$ as $g_s^{-1/2}$ (somewhere in between 1 and g_s^{-1}), and $|\partial_z^2 \Pi|^2 / [\int \Omega_X \wedge \bar{\Omega}_X]$ as $\mathcal{O}(1)$. This reproduces the mass estimate (M_{KK}^3/M_s^2) of the complex structure moduli in [9], passing the first sanity check.¹²

The physical mass of Kähler moduli $m_{T\bar{T}}^2$, on the other hand, is estimated by

$$\begin{aligned} m_{T\bar{T}}^2 &\sim \left[\frac{1}{i \int_X \Omega \wedge \bar{\Omega}} \frac{g_s}{2} \frac{1}{\text{Re}(\langle T \rangle)^3} \right] \left[\frac{\text{Re}(\langle T \rangle)^2}{3} \right]^2 \frac{\left| \sum_{i=1,2} a_i(z) (N_i d_i)^2 e^{-d_i \langle T \rangle} \right|^2}{[(4\pi) \text{Re}(\langle T \rangle)]^3} M_{\text{Pl}}^2 \\ &\sim \frac{(2\pi)^4 M_{\text{Pl}}^2}{(4\pi)^3 \text{Re}(\langle T \rangle)^2} \sim \left[\frac{(2\pi)^2}{4\pi \ell_s \sqrt{\langle \omega \rangle}} \right]^2 \sim [\pi M_{KK}]^2; \end{aligned} \quad (40)$$

we ignored the complex structure moduli dependence of a_i 's in (9), so that the mixing terms disappear, and $m_{T\bar{T}}^2 \sim e^{K/M_{\text{Pl}}^2} |K^{\bar{T}T} \partial_{\bar{T}}^2 W|^2$ is used for the estimate. This result is acceptable, in that the physical mass should not be higher than the Kaluza–Klein scale (where 4D $\mathcal{N} = 1$ super Yang–Mills is not a good approximation). Thus, we do not find anything counter intuitive in the result of the physical mass $m_{T\bar{T}}^2$ of the Kähler moduli. In reality, the value of $|\sum_i a_i e^{-d_i \langle T \rangle}|^2 / [(i \int_X \Omega \wedge \bar{\Omega}) \cdot (2/g_s)]$ —a factor treated as $\mathcal{O}(1)$ in the second line—will be smaller than unity because of the exponential factors (see the next section for more). Depending on the exponential factors, diagonal entries of the physical mass matrix may be larger in $m_{T\bar{T}}^2$ or in $m_{z\bar{z}}^2$; they can also be comparable in size.

The discussion so far indicates that $|W^{(T)}| \ll |W^{(\text{cpx})}|$ generically (in the moduli space) in Type IIB/F-theory compactifications, unless a dedicated condition like (6) is imposed for

¹²The physical mass $m_{z\bar{z}}^2$ comes out to be hierarchically smaller than M_{Pl}^2 because of $e^{\langle K \rangle / M_{\text{Pl}}^2} \ll 1$ in the convention in this article. Due to the Kähler–super Weyl transformation in 4D $\mathcal{N} = 1$ supergravity (e.g., arbitrariness of the normalization of Ω_X), both the superpotential and the Kähler potential need to be included in order to obtain physical $m_{z\bar{z}}^2$.

phenomenology. However, $m_{T\bar{T}}^2$ can be larger than $m_{z\bar{z}}^2$, in the case exponential factors are not too small, because of $M_{\text{Pl}}^2 \langle K^{\bar{T}T} \rangle \sim (\text{Re} \langle T \rangle)^2 \gg 1$. In the large complex structure region of the moduli space, though, the physical mass $m_{z\bar{z}}^2$ may also be enhanced by $M_{\text{Pl}}^2 K^{\bar{z}z} \gg 1$.

1-loop threshold corrections to the gauge coupling have been computed explicitly for some set-ups of string compactifications. Based on such known forms of the threshold corrections, it is reasonable to assume that $\langle \partial_z a(z) \rangle$ is not much different from $\langle a(z) \rangle$, both being dimensionless values of order unity. For this reason, $\langle \partial_T \partial_z W \rangle$ in the off-diagonal entries of the mass matrix (9) is not expected to be much smaller than $\langle \partial_T^2 W \rangle$ in the diagonal entry. We have used this expectation already in the discussion on reheating process at the end of the previous section.

4 Robust Estimate of Decompactification Constraint

Having studied the prefactor of the effective superpotential from gaugino condensation, let us now study how the Kähler moduli stabilization requirement (which we discussed in section 2) constrains the possibility of inflationary process involving complex structure moduli. In this section and in the rest of this article, we stick to systems with effectively only one Kähler modulus field for simplicity, with the no-scale type Kähler potential (see footnote 5), and use the racetrack superpotential from two gaugino condensations.

Racetrack superpotential has long been studied in the literature for volume stabilization [15, 21]. In our context, where gaugino condensation of $\text{SU}(N_1) \times \text{SU}(N_2)$ contributes to the racetrack superpotential,

$$W^{(T)} = \frac{M_{\text{Pl}}^3}{[(4\pi)\text{Re} \langle T \rangle]^{3/2}} \left(a_1 N_1^2 e^{-\frac{2\pi}{N_1} T} + a_2 N_2^2 e^{-\frac{2\pi}{N_2} T} \right), \quad (41)$$

where a_1, a_2 are dimensionless and may depend on the complex structure moduli.

4.1 Barrier Height of Volume Stabilization Potential

We begin with a review of important properties of this superpotential. In this section 4.1, we take a close look at an effective theory where all the complex structure moduli fields are replaced by their vacuum values. This is to take

$$W^{(\text{tot})} = W^{(T)}(T) + W_0, \quad (42)$$

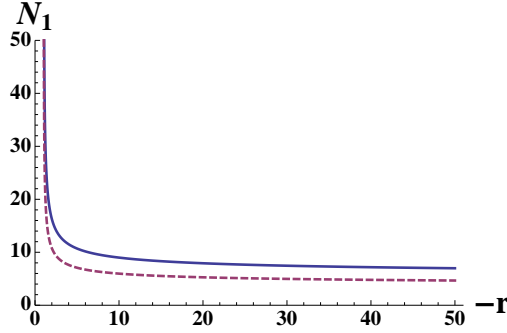


Figure 1: This is a contour plot of the vacuum value $\langle T \rangle$ on the $(-r, N_1)$ plane; the curve above (solid) is the contour of $\langle T \rangle = 25$, and the one below (dashed) that of $\langle T \rangle = 10$.

and the Kallosh–Linde tuning implies that

$$W_0 = -\langle W^{(T)} \rangle. \quad (43)$$

From a practical perspective, it is best to study how the scalar potential for the Kähler modulus T changes, for different vacuum values $\langle T \rangle$. With a simplifying ansatz that

$$N_1 = N_2 + 1, \quad (44)$$

the vacuum value $\langle T \rangle$ is determined by

$$\langle T \rangle = \frac{N_1(N_1 - 1)}{2\pi} \ln \left[-r \frac{N_1 - 1}{N_1} \right] \quad (45)$$

as a function of N_1 and a ratio $r := \langle a_2/a_1 \rangle$, and then W_0 is given by

$$W_0 = -\langle W^{(T)} \rangle = -\frac{M_{\text{Pl}}^3}{[(4\pi)\text{Re}\langle T \rangle]^{3/2}} \langle a_1 \rangle N_1 \left(-\frac{\langle a_1 \rangle}{\langle a_2 \rangle} \frac{N_1}{N_2} \right)^{N_2}. \quad (46)$$

Figure 1 shows how the required value of N_1 changes for different values of $r = \langle a_2/a_1 \rangle$ in order to achieve a given value of $\text{Re}\langle T \rangle$.

In supersymmetric extensions of the Standard Model with supersymmetry breaking around the TeV—a few hundred TeV, the unified gauge coupling constant is approximately $1/\alpha_{\text{GUT}} \sim 25$; if there are $\text{SU}(5)_{\text{GUT}}$ -charged particles in the light spectrum (e.g., in gauge mediated supersymmetry breaking scenario), however, the unified coupling constant may be stronger. It is not crazy to think also of $1/\alpha_{\text{GUT}} \sim 10$ for this reason. When the gauge groups of the

supersymmetric Standard Models originate from 7-branes wrapped on a 4-cycle in a Calabi–Yau 3-fold, such values of $1/\alpha_{\text{GUT}} = 4\pi/g_{\text{GUT}}^2$ can be used as the vacuum value of $\text{Re} \langle T \rangle$. The two contours in Figure 1 are drawn for $\langle T \rangle = 25$ and $\langle T \rangle = 10$ for this reason.¹³

It is worth noting that, for any choice of the parameters $\langle a_2/a_1 \rangle$, $N_{1,2}$ of the effective theory, the potential barrier height of the Kähler modulus in

$$V_{\text{racetrack}}(T) := \frac{1}{|a_1|^2} e^{-3 \ln[\text{Re}(T)]} \left[K^{\bar{T}T} |D_T W^{(\text{tot})}|^2 - 3 \left| \frac{W^{(\text{tot})}}{M_{\text{Pl}}} \right|^2 \right] \quad (48)$$

using $W^{(\text{tot})}$ in (42) is bounded from above by

$$\begin{aligned} [V_{\text{racetrack}}]_{\text{barrier}} &\approx \frac{1}{[\text{Re} \langle T \rangle]^3} \frac{[\text{Re} \langle T \rangle]^2}{M_{\text{Pl}}^2} \frac{M_{\text{Pl}}^6}{(4\pi)^3 [\text{Re} \langle T \rangle]^3} \times \left| (2\pi/N_i) N_i^2 e^{-\frac{2\pi}{N_i} \langle T \rangle}, \text{s} \right|^2, \\ &\approx \frac{M_{\text{Pl}}^4}{(4\pi)^3 [\text{Re} \langle T \rangle]^4} \left| (2\pi N_i) e^{-\frac{2\pi}{N_i} \langle T \rangle}, \text{s} \right|^2 \approx \frac{M_{KK}^4}{4\pi} \left| (2\pi N_i) e^{-\frac{2\pi}{N_i} \langle T \rangle}, \text{s} \right|^2. \end{aligned} \quad (49)$$

It cannot exceed the energy scale $(M_{KK})^4$ by much, obviously from construction, and is further suppressed by exponential factors.

Figure 2 shows the potential $V_{\text{racetrack}}(T)$ for a couple of different choices of (r, N_1) satisfying $\text{Re} \langle T \rangle = 25$. The shape of the volume stabilizing potential $V_{\text{racetrack}}(T)$ and the height of the barrier are very sensitive to the choice of (r, N_1) . This high sensitivity of the barrier height is also an unavoidable consequence in the racetrack potential. To see this, note that the height of the potential barrier, $[V_{\text{racetrack}}]_{\text{barrier}}$, is qualitatively explained by the two factors: one is

$$\frac{(M_{KK})^4}{4\pi} \simeq \frac{M_{\text{Pl}}^4}{(4\pi)^3 (\text{Re} \langle T \rangle)^4} \simeq 1.3 \times 10^{-9} \left(\frac{25}{\text{Re} \langle T \rangle} \right)^4 M_{\text{Pl}}^4, \quad (50)$$

¹³ It is possible that there are a couple of integer factors entering various formulas we have used so far. First, depending on the intersection ring of a Calabi–Yau 3-fold for Type IIB compactification, $M_{\text{Pl}}^2 \ell_s^2 = (4\pi) M_0 \langle \omega \rangle^3 / g_s^2$ with $M_0 \geq 1$. Secondly, depending on which topological cycle the GUT 7-branes are wrapped on, $1/\alpha_{\text{GUT}} = M_1 \langle \omega \rangle^2 / g_s$. This factor also affects the Kaluza–Klein scale on the GUT 7-brane: $M_{KK; \text{GUT}} = 1/[\ell_s \sqrt{\langle \omega \rangle} M_2]$. Combining them together,

$$\left(\frac{M_{\text{Pl}}}{M_{KK; \text{GUT}}} \right)^2 = M_0 M_2^2 (4\pi) [\text{Re} \langle T \rangle]^2 = M_0 (M_2/M_1)^2 \frac{4\pi}{[\alpha_{\text{GUT}}]^2}. \quad (47)$$

Because of the fudge factor $M_0(M_2/M_1)^2$, it does not immediately run into inconsistency to choose $M_{KK; \text{GUT}} \simeq (\text{a few}) 10^{16} \text{GeV}$, while taking $1/\alpha_{\text{GUT}} \sim 10$, for example. One should keep in mind, however, that there is such a tension. See also footnote 5. For the parameter fitting in the case of F-theory compactifications, see [2].

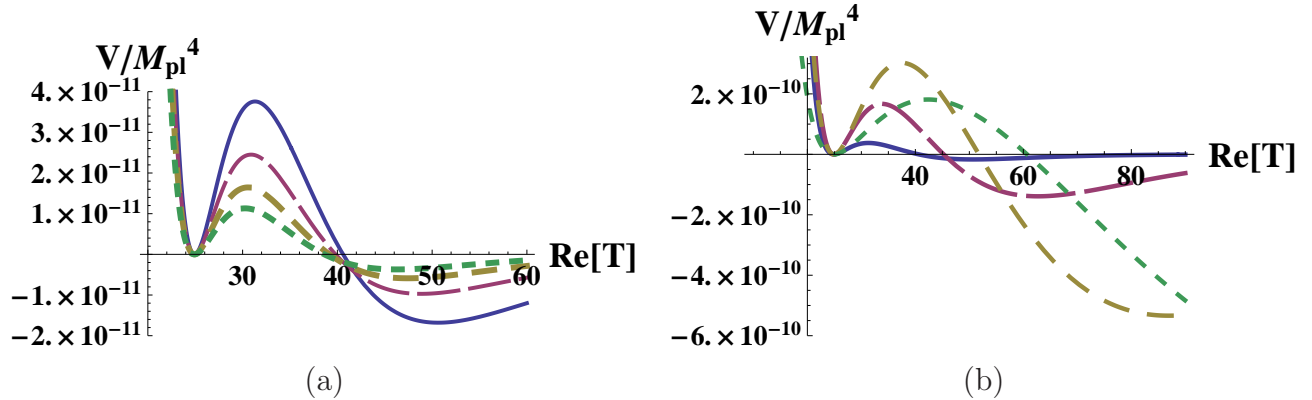


Figure 2: The potential $V_{\text{racetrack}}(T)$ for a couple of different choices of $r := \langle a_2/a_1 \rangle$ and N_1 that lead to $\langle T \rangle = 25$. **Panel (a):** the curves shown in solid, long dashed, dashed and dotted lines are for parameters $(r, N_1) = (-1.05, 68)$, $(-1.06, 62)$, $(-1.07, 57)$ and $(-1.08, 53)$, respectively. The larger the value of r is, the larger the height of the potential barrier becomes. **Panel (b):** the potential for the values of $(r, N_1) = (-1.05, 68)$, $(-1.02, 118)$, $(-1.007, 238)$ and $(-1.002, 626)$ is shown in a solid, long dashed, dashed and dotted line, respectively. The second (negative energy) minimum deepens for larger r .

and the other is from the extra exponential factors in (49); the latter is evaluated to be¹⁴

$$\left| (2\pi N_1) e^{-\frac{2\pi}{N_1} \langle T \rangle} \right|_{\langle T \rangle = 25}^2 \simeq \begin{cases} 1800 & (r, N_1) \simeq (-1.05, 68), \\ 0.002 & (r, N_1) \simeq (-1.67, 20). \end{cases} \quad (51)$$

For the value of $r = -\langle a_2/a_1 \rangle$ close to -1 , where $\ln[-r] \simeq 0$, the value of N_1 for a given $\langle T \rangle$ changes rapidly, as seen in (45) or in Figure 1, and hence the exponential factor becomes small very quickly as the value of r differs from -1 . For larger value of $|r|$, N_1 does not change much, so that the rapid decrease in the barrier height slows down (Figure 3). To summarize, the potential barrier $[V_{\text{racetrack}}]_{\text{barrier}}/M_{\text{Pl}}^4$ is highly sensitive to the choice of the parameter $r = -\langle a_2/a_1 \rangle$, because the value of $\langle T \rangle/N_1$ depends very much on r , and further because it is exponentiated.

¹⁴In reality, the two factors—(50) and (51) combined—almost explains the hierarchy $[V_{\text{racetrack}}]_{\text{barrier}} \ll M_{\text{Pl}}^4$ obtained numerically, but unaccounted hierarchy still remains. For example, for the choice $(r, N_1) \simeq (-1.05, 68)$, we found $[V_{\text{racetrack}}]_{\text{barrier}} \sim 10^{-11} \times M_{\text{Pl}}^4$ numerically [Figure 2(a)], whereas the two factors (50, 51) combined predicts $10^{-9} \times 10^3$. Similarly for $(r, N_1) \simeq (-1.67, 20)$, the numerical results was $[V_{\text{racetrack}}]_{\text{barrier}}/M_{\text{Pl}}^4 \simeq 10^{-15}$ [Figure 3], which is still smaller than the estimate $10^{-9} \times 10^{-3}$. The combination of (50) and (51) tends to overestimate $[V_{\text{racetrack}}]_{\text{barrier}}/M_{\text{Pl}}^4$ presumably because there is cancellation between the two exponential factors, and also because the barrier height should have been estimated by using a value of T somewhat larger than $\langle T \rangle$.

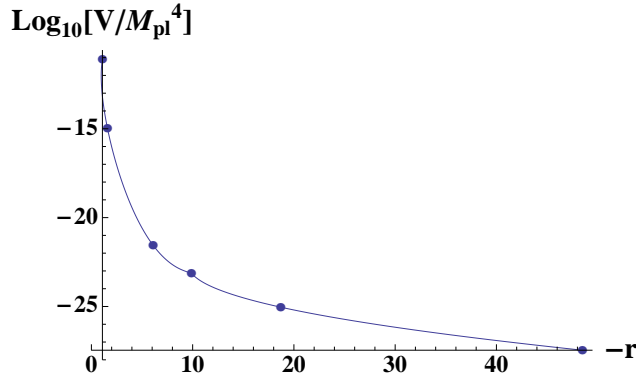


Figure 3: The potential barrier height $[V_{\text{racetrack}}]^{\text{barrier}}/M_{\text{Pl}}^4$ changes by orders of magnitude for different choices of (r, N_1) . The data points are for $(r, N_1) \simeq (-1.1, 50)$, $(-1.67, 20)$, $(-6.1, 10)$, $(-9.9, 9)$, $(-18.7, 8)$, and $(-48.5, 7)$.

The potential barrier height can remain close to M_{KK}^4 , when the value of $\ln[-r] = \ln[-\langle a_2/a_1 \rangle]$ is very small, obviously from the discussion above. Even in an extreme choice¹⁵ $(r, N_1) \simeq (-1.007, 238)$, however, the barrier height remains to be roughly 9.5 orders of magnitude below M_{Pl}^4 , or equivalently, $[V_{\text{racetrack}}]^{\text{barrier}} \sim (10^{16} \text{ GeV})^4$. When the parameter r is not as finely tuned as above, the energy scale of the barrier height, $([V_{\text{racetrack}}]^{\text{barrier}})^{1/4}$, is already five or six orders of magnitude below the Planck scale M_{Pl} .

We consider that it is a generic consequence of the racetrack superpotential from 4D gaugino condensations in the regime $\langle T \rangle \approx \mathcal{O}(10)$ or somewhat larger, even with the Kallosh–Linde tuning (6), that the height of the volume stabilizing barrier is no higher than M_{KK}^4 , and it generically comes with extra exponential suppression. The exponential suppression is mitigated only with very dedicated choice of parameters $(\langle a_2/a_1 \rangle, N_1)$. We have also done the same analysis for $\langle T \rangle = 10$ as well, and found similar results; $[V_{\text{racetrack}}]^{\text{barrier}}$ can only be as high as $(\text{a few} \times 10^{16} \text{ GeV})^4$ for an extreme choice of $(\langle a_2/a_1 \rangle, N_1)$. It is likely that the same argument holds true for any other mechanism for $W^{(T)}$, as long as it relies on non-perturbative dynamics in the 4D effective gauge theory below the Kaluza–Klein scale.

Certainly one cannot use the barrier height of the potential $V_{\text{racetrack}}(T)$ alone to exclude some kinds of inflation models, since the potential $V_{\text{racetrack}}(T)$ in (48) is not the same as the scalar potential of the full theory (2). It has been accepted as a rule of thumb, however,

¹⁵For r even closer to -1 , however, the potential barrier height is not as high as in the case with $r = -1.007$; see Figure 2 (b). We have not looked into the details of what is going on.

that the volume stabilization may be in jeopardy, if the vacuum energy during inflation is comparable to the potential barrier height of $V_{\text{racetrack}}(T)$.

Therefore we have an important lesson, under this “rule”. In the regime of moderately large volume, i.e., $\text{Re}\langle T \rangle \sim 10$ or somewhat larger, it is very hard to accommodate inflation with the energy density $\rho \sim (10^{16} \text{ GeV})^4$ or larger; this energy density corresponds to the tensor-to-scalar ratio that can be probed in a near future (10^{-1} – 10^{-2} or so). Depending on the value of the tensor-to-scalar ratio, one might be motivated to think of extra light matter particles (so that the unified gauge coupling is stronger) and/or Kaluza–Klein scale higher than the energy scale of apparent gauge coupling unification, or to throw away the scheme of supersymmetric unification altogether. Note that this statement is very robust,¹⁶ because it only refers to the energy density during inflation, not to details of inflation models.

In the analysis of section 4.2, we use the following parameter set (like in [10]):

$$N_1 = 68, \quad N_2 = 67, \quad \langle a_2/a_1 \rangle \simeq -1.05041, \quad (52)$$

when $\langle T \rangle = 25$ and $[V_{\text{racetrack}}]^{\text{barrier}} \simeq 4 \times 10^{-11} \times M_{\text{Pl}}^4$ (the tuning (6) is understood). This choice of parameters is less extreme than $(\langle a_2/a_1 \rangle, N_1) = (-1.007, 238)$, so we expect to derive a more robust constraint on the complex structure deformation allowed during the inflationary process. The other (and more important) reason is that the potential $V_{\text{racetrack}}(T)$ develops a deep negative energy minimum in the region $\text{Re}(T) \gg \langle T \rangle = 25$, if $\langle a_2/a_1 \rangle$ is chosen to be close to the value -1.007 for the highest $[V_{\text{racetrack}}]^{\text{barrier}}$ possible (see Figure 2(b)). We need to be worried about the quantum tunneling in that case. The choice above is more moderate from that perspective. Although this quantum stability argument should also be included in the discussion on the tensor-to-scalar ratio above, we do not try to conduct quantitative analysis in this article.

4.2 Phenomenological Study of Volume Destabilization using $V_{\text{racetrack}}$

If we are to seek for a case where the complex structure moduli sector z plays a non-negligible role during inflation, then $W^{(\text{cpx})}$ may be different from its vacuum value $\langle W^{(\text{cpx})} \rangle$. When the deviation $W^{(\text{cpx})} - \langle W^{(\text{cpx})} \rangle$ is too large, the last two terms of (5) [i.e., the second term of (2)] may no longer be positive, and/or the Kähler moduli may start to decompactify.

¹⁶ It should be reminded, though, that our numerical study is only for a pair of gaugino condensations, and a relation (44) is imposed just to keep the presentation simple. Details in footnote 13 should also be taken into account.

To study the constraint from volume destabilization, one should use the full scalar potential (2, 5), but it is messy, complicated, and even worse, depends very much on the choice of Calabi–Yau geometry for compactification. In order to extract as robust lessons as possible, we begin with a little phenomenological approach instead. That is a) to ignore the first term of (2), and b) to deal with a limited number of parameters capturing all the influence of complex structure moduli evolution entering the remaining term—the second term—in (2). We bring in a bit of guess work in deriving the destabilization constraint, in order to overcome the disadvantage associated with (a). It takes an extra effort to carry out model-specific analysis using the full scalar potential (without the guess work); discussion in section 5.2 can be regarded as a necessary first step.

There are only three different ways the field value of complex structure moduli z enters into the second term of (2). First, the total superpotential $W^{(\text{tot})}$ contains $W^{(\text{cpx})}(z)$ which depends on z , and secondly, we should also expect z -dependence in a_i 's of the racetrack superpotential. Although the overall factor $e^{\mathcal{K}^{(\text{cpx})}}$ also introduces z -dependence, this overall factor is more or less irrelevant to the study of instability toward decompactification $\text{Re}(T) \rightarrow \infty$. The first effect is parametrized by $\delta\widetilde{W}_{\text{eff.}}^{(\text{cpx})}$ in

$$W^{(\text{tot})} = \frac{M_{\text{Pl}}^3}{[(4\pi)\text{Re}\langle T \rangle]^{3/2}} \left(\sum_i a_i N_i^2 e^{-\frac{2\pi}{N_i}T} \right) - \langle W^{(T)} \rangle + \frac{M_{\text{Pl}}^3}{\sqrt{4\pi}} \delta\widetilde{W}_{\text{eff.}}^{(\text{cpx})}; \quad (53)$$

the value of $W^{(\text{cpx})}(z)$ is split into $W_0 + M_{\text{Pl}}^3/\sqrt{4\pi} \times \delta\widetilde{W}_{\text{eff.}}^{(\text{cpx})}$, the vacuum value and deformation from it during the inflation and/or reheating process. We implement the other z -dependence by dealing with a_2/a_1 as another parameter (which may also be different from its vacuum value $\langle a_2/a_1 \rangle$).

We study how much $\delta\widetilde{W}_{\text{eff.}}^{(\text{cpx})}/a_1$ and a_2/a_1 can be changed without distorting the T -dependent potential $V_{\text{racetrack}}(T)$ too much; the superpotential $W^{(\text{tot})}$ in (53) is used for $V_{\text{racetrack}}(T)$ instead of $W^{(T)} + W_0$, so that the potential $V_{\text{racetrack}}(T)$ is now roughly equal to the second term of (2) with the irrelevant overall factor $e^{\mathcal{K}^{(z)}}$ stripped off. As we have announced at the end section 4.1, we choose the value $N_1 = 68$, $N_2 = 67$ and $r = \langle a_2/a_1 \rangle = -1.05041$, so that $\langle T \rangle = 25$ and the Kähler modulus is stabilized for the vacuum value of complex structure moduli $z = \langle z \rangle$. Although it is desirable to study the deformation of $V_{\text{racetrack}}(T)$ for the complex two-dimensional parameter space of deformation, $(\delta\widetilde{W}_{\text{eff.}}^{(\text{cpx})}/a_1, a_2/a_1)$, we will carry out the study only along three real 1-parameter deformations in the parameter space in the following.

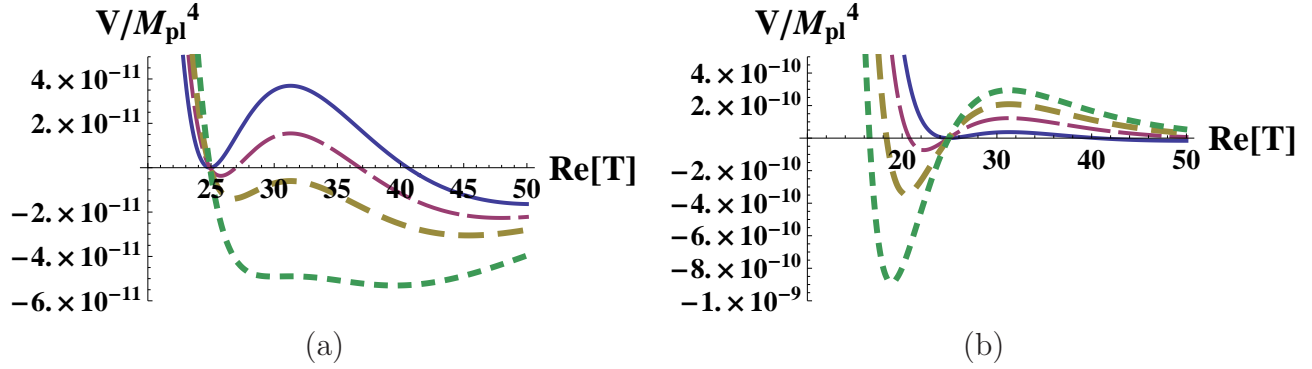


Figure 4: The potential $V_{\text{racetrack}}(T)$ with $W^{(\text{tot})}$ in (53) for various values of $\delta\widetilde{W}_{\text{eff}}^{(\text{cpx})}(z)/a_1$; the vacuum parameters in (52) are used here, and the other deformation parameter a_2/a_1 is held fixed at its vacuum value in this figure. **Panel (a):** the four curves from top to bottom (solid, long dashed, dashed and dotted) correspond to the deformation parameters $\delta\widetilde{W}_{\text{eff}}^{(\text{cpx})}/a_1 = 0, -0.0005, -0.001$ and -0.002 , respectively. **Panel (b):** the potential for the deformation $\delta\widetilde{W}_{\text{eff}}^{(\text{cpx})}/a_1 = 0, 0.002, 0.004$ and 0.006 are shown in the solid, long dashed, dashed and dotted curves, respectively.

(i) variation of $W^{(\text{cpx})}(z)/a_1$

Figure 4 (a, b) shows how $V_{\text{racetrack}}(T)$ changes as we change the value of $(\delta\widetilde{W}_{\text{eff}}^{(\text{cpx})})/a_1$ while keeping a_2/a_1 fixed at the vacuum value $r = \langle a_2/a_1 \rangle$. Starting from the vacuum value $\delta\widetilde{W}_{\text{eff}}^{(\text{cpx})}/a_1 = 0$ (i.e., $W^{(\text{cpx})}(z) = W_0 = -\langle W \rangle^{(T)} \simeq -0.00122 \times a_1 M_{\text{Pl}}^3$) and adding deformation $\delta\widetilde{W}_{\text{eff}}^{(\text{cpx})}$ in the negative real-valued region, we see that the barrier in $V_{\text{racetrack}}(T)$ almost disappears by the time $\delta\widetilde{W}_{\text{eff}}^{(\text{cpx})}/a_1 \simeq -0.001$. Certainly the Kähler moduli dependence of the full scalar potential (2) is not the same as $V_{\text{racetrack}}(T)$, because the first term provides uplift¹⁷ at least around $T \sim \langle T \rangle = 25$. With some uplift component added to the T -dependent potential in Figure 4 (a), it is hard to imagine, however, that the uplift term drastically improves the potential for volume stabilization (even against quantum tunneling) for the range of deformation $\delta\widetilde{W}_{\text{eff}}^{(\text{cpx})}/a_1 \sim -0.001$. Therefore, it will not be terribly bad to take this value as an estimate of the limit on how much $\delta\widetilde{W}_{\text{eff}}^{(\text{cpx})}$ can change without jeopardizing the Kähler moduli stabilization.

Perturbing the value of $\delta\widetilde{W}_{\text{eff}}^{(\text{cpx})}$ in the real positive region instead, we see in Figure 4 (b)

¹⁷Remember that the vacuum energy $V(z, T)$ vanishes at the vacuum value $z = \langle z \rangle$, $T = \langle T \rangle$ under the Kallosh–Linde tuning (6), and the potential energy must be positive for any small perturbation from $z = \langle z \rangle$ and $T = \langle T \rangle$.

that the potential barrier of $V_{\text{racetrack}}(T)$ becomes higher, but the local minimum around $T \sim 25$ gets deeper and deeper at the same time. When the value of $\delta\widetilde{W}_{\text{eff.}}^{(\text{cpx})}/a_1$ is around $+0.004$, the potential dip in $V_{\text{racetrack}}(T)$ is much more than the barrier height in their absolute values; the uplifting contribution from the first term of (2) will push this dip at least above the zero energy density of $V(z, T)$. This means that the T -dependence of the full potential $V(z, T)$ in (2) is different from that of $V_{\text{racetrack}}(T)$ considerably, and we cannot say the Kähler moduli T remains to be stabilized without studying the full potential (2) for such a value of $\delta\widetilde{W}_{\text{eff.}}^{(\text{cpx})}$. This argument does not rule out a possibility that the first term of (2) provides just enough uplift around $T \sim 20$ so that $\rho = V(T, z)$ becomes barely positive, but not as high as the potential barrier around $T \sim 30$; we do not have a concrete idea of how to coordinate the first and second terms in (2) for that to happen, however.

For these reasons, and in this meaning, we see that the volume stabilization in $V_{\text{racetrack}}(T)$ remains to be reliable as long as the value of $W^{(\text{cpx})}(z)$ differs from its vacuum value W_0 within the range

$$-0.001 \lesssim \delta\widetilde{W}_{\text{eff.}}^{(\text{cpx})}/a_1 \lesssim 0.004. \quad (54)$$

This estimate of the limit can be read as that of the deformation in $a_1^{-1} \int_X G \wedge \Omega = \sum_a (n_a^R - \tau n_a^{NS}) \Pi_a / a_1$, since we chose the normalization of $\delta\widetilde{W}_{\text{eff.}}^{(\text{cpx})}$ that way in (53).

(ii) variation of $a_2/a_1(z)$

Figure 5 shows how $V_{\text{racetrack}}(T)$ changes when the value of a_2/a_1 is different from its vacuum value $r = \langle a_2/a_1 \rangle \simeq -1.05041$ instead, while the value of $W^{(\text{cpx})}(z)$ somehow remains to be W_0 . We can use the numerical results in the figure, to set a limit

$$-1.0550 \lesssim a_2/a_1 \lesssim -1.0400 \quad (55)$$

for the same reason as in the analysis of changing $\delta\widetilde{W}_{\text{eff.}}^{(\text{cpx})}/a_1$. The potential $V_{\text{racetrack}}(T)$ is highly sensitive to the value of a_2/a_1 , when the value of $W^{(\text{cpx})}(z)/a_1$ is held fixed, and there is not much room around the vacuum value $\langle a_2/a_1 \rangle \simeq -1.05041$. This result is not hard to imagine from the discussion in section 4.1, because of the high sensitivity of the potential $V_{\text{racetrack}}(T)$ on the vacuum value $\langle a_2/a_1 \rangle$. If we replace the vacuum parameters in (52) by more negative $r = \langle a_2/a_1 \rangle$ and smaller N_1 , we expect larger range of deformation in a_2/a_1 from the new vacuum value will be allowed, than in (55); this comes at the cost of limiting the energy density during inflation from above, however.

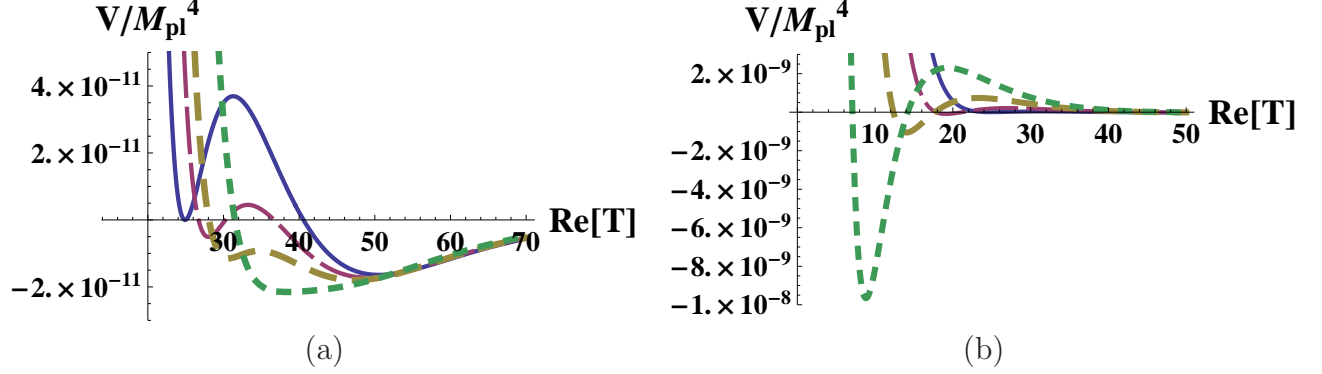


Figure 5: The potential $V_{\text{racetrack}}(T)$ with $W^{(\text{tot})}$ in (53) for various values of a_2/a_1 ; the value of $W^{(\text{cpx})}(z)$ is fixed at W_0 in this figure. **Panel (a):** the four curves (solid, long dashed, dashed and dotted) are for $a_2/a_1 \simeq -1.0504, -1.0530, -1.0550$ and -1.0600 , respectively. **Panel (b):** the parameter value is set at $a_2/a_1 = -1.0504, -1.0450, -1.0400$ and -1.0350 in the curves drawn in the solid, long dashed, dashed and dotted lines, respectively. The potential $V_{\text{racetrack}}(T)$ drawn in the solid line in (a) and (b) are the same, the one at the vacuum $a_2/a_1 = \langle a_2/a_1 \rangle$.

(iii) variation of both $\delta\widetilde{W}_{\text{eff.}}^{(\text{cpx})}/a_1$ and $a_2/a_1(z)$

We have so far searched only along two real-valued 1-parameter deformations in the phenomenological parameter space $(\delta\widetilde{W}_{\text{eff.}}^{(\text{cpx})}/a_1, a_2/a_1)$ of $V_{\text{racetrack}}(T)$. This leaves a room for some combination of changes in $W^{(\text{cpx})}(z)$ and in a_2/a_1 so that the potential barrier remains in $V_{\text{racetrack}}(T)$. There is no obvious strategy to look for such a coordinated changes, or to claim their absence. We just try one more, a 1-parameter deformation in $W^{(\text{cpx})}$ and a_2/a_1 so that $V_{\text{racetrack}}(T)$ remains to have vanishing energy at the local minimum around $T \sim \langle T \rangle \simeq 25$ (the energy density of the full potential (2) is positive); this corresponds to focus on a subspace of $(\delta\widetilde{W}_{\text{eff.}}^{(\text{cpx})}/a_1, a_2/a_1)$ satisfying a relation

$$-\frac{M_{\text{Pl}}^3 a_1 N_1}{(4\pi \text{Re} \langle T \rangle)^{3/2}} \left(-\frac{a_1 N_1}{a_2 N_2} \right)^{N_2} = W^{(\text{cpx})} = W_0 + \frac{M_{\text{Pl}}^3}{\sqrt{4\pi}} \delta\widetilde{W}_{\text{eff.}}^{(\text{cpx})}, \quad (56)$$

which generalizes (43).

As in Figure 6 (a), for the choice of a_2/a_1 closer to zero than the vacuum value $\langle a_2/a_1 \rangle \simeq -1.0504$ [i.e., $\delta\widetilde{W}_{\text{eff.}}^{(\text{cpx})}/a_1 \leq 0$], the potential barrier $V_{\text{racetrack}}(T)$ becomes higher (read the caption carefully), but the second minimum (at larger value of T) also gets deeper more rapidly. The Kähler moduli field T may remain marginally stable in $V_{\text{racetrack}}(T)$ for $a_2/a_1 = -1.035$, but it will not be against quantum tunneling for a value even closer to zero. Thus,

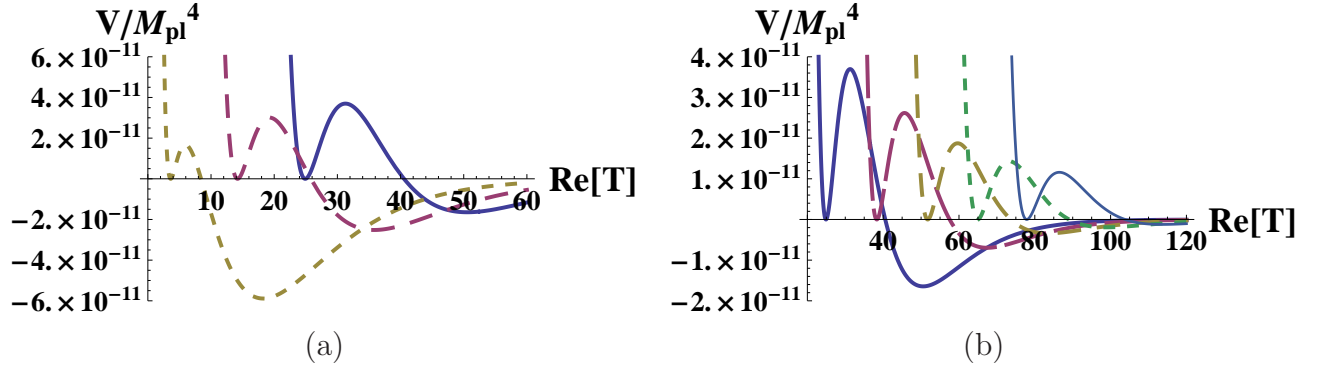


Figure 6: The potential $V_{\text{racetrack}}(T)$ with $W^{(\text{tot})}$ in (53) with different sets of deformation parameters $(\delta\widetilde{W}_{\text{eff}}^{(\text{cpx})}/a_1, a_2/a_1)$ satisfying the relation (56). **Panel (a):** $V_{\text{racetrack}}(T)$ is drawn for the deformation parameters $(0, -1.0504)$, $(-0.007, -1.0350)$ and $(-0.027, -1.0200)$ in the solid, dashed and dotted line, respectively, after multiplied by 1, 10^{-1} and 10^{-2} , respectively; this means, for example, that the local maximum of the potential for $(-0.027, -1.0200)$ is about $2 \times 10^{-9} \times M_{\text{Pl}}^4$. **Panel (b):** the solid, long dashed, dashed, dotted and thin solid lines show the potential $V_{\text{racetrack}}(T)$ for the sets of deformation parameters $(0, -1.0504)$, $(0.0031, -1.07)$, $(0.0040, -1.09)$, $(0.0042, -1.11)$ and $(0.0043, -1.13)$, respectively, multiplied by a factor 1, 10, 10^2 , 10^3 and 10^4 , respectively; this means, for example, that the local maximum of $V_{\text{racetrack}}(T)$ is about $10^{-15} \times M_{\text{Pl}}^4$ for the deformation $(0.0043, -1.13)$.

this destabilization argument sets a limit in the deformation satisfying (56) as follows:

$$-0.007 \lesssim \delta\widetilde{W}_{\text{eff}}^{(\text{cpx})}/a_1, \quad a_2/a_1 \lesssim -1.035. \quad (57)$$

With the coordinated deformation in (56), certainly the destabilization limit above has been relaxed a bit from (54, 55), but not much. For deformation in the other direction, however, the second minimum in the potential $V_{\text{racetrack}}(T)$ becomes less and less pronounced for even more negative value of a_2/a_1 relatively to the vacuum value -1.0504 , as one can see in Figure 6 (b). There may be no danger of destabilization for such a change, and the lower bound on a_2/a_1 disappears. This considerably relaxes the constraint $-1.0550 \lesssim a_2/a_1$ obtained earlier (only the $W^{(\text{cpx})} \leq 0$ region is probed under the relation (56)). The potential barrier height is also reduced considerably at the same time, however, for such a deformation in $(\delta\widetilde{W}^{(\text{cpx})}/a_1, a_2/a_1)$; see the caption of the figure. In order to support inflation at high energy scale, that may not be appropriate.

5 Constraint on Complex Structure Deformation during/after Inflation

In the previous section, we have studied how much complex structure moduli can be deformed from their vacuum values during the inflation/reheating process by (I) using parameters representing the effects of complex structure deformation, and (II) studying the potential $V_{\text{racetrack}}(T)$ (rather than the full scalar potential (2)). In sections 5.1 and 5.2, we make it clear what the constraint on the deformation means in Type IIB Calabi–Yau orientifold compactifications, and at the same time, we carry out necessary technical preparation so that the analysis using the full scalar potential is possible. In section 5.3, a few ideas of how to avoid/satisfy the constraint are discussed.

5.1 An Attempt of Implementing KYY in Type IIB Orientifolds

5.1.1 A Brief Note on Mirror-Quintic

We use the “mirror-quintic” as a Calabi–Yau 3-fold X for Type IIB orientifold compactification and something similar to the mirror quintic, for the presentation purpose in sections 5.1 and 5.2; much of the statements will remain valid for broader class of Type IIB Calabi–Yau orientifolds and for F-theory compactifications, however. To be more precise, we use the superpotential $W^{(\text{cpx})} = W_{GVW}$ and $K^{(\text{cpx})}/M_{\text{Pl}}^2 = -\ln[i \int_X \Omega \wedge \overline{\Omega}]$ based on the mirror-quintic, while we still continue to use only one Kähler modulus chiral multiplet T and the no-scale type Kähler potential (see footnote 5). Obviously the statements in sections 5.1 and 5.2 should not be taken as precise, rigorous results about compactifications on the mirror quintic X ; the following discussion is rather meant to be a test study of general properties in Type IIB or F-theory compactifications.

Before getting started, we leave a brief summary of known facts about the mirror-quintic that are used in the following discussion.

The mirror-quintic Calabi–Yau 3-fold X has only one complex structure modulus. It is regarded as a crepant resolution of $\mathbb{Z}_5 \times \mathbb{Z}_5 \times \mathbb{Z}_5$ orbifold of a geometry given by

$$\left\{ [X_1 : \cdots : X_5] \in \mathbb{P}^4 \mid \sum_{i=1}^5 (X_i)^5 - 5\psi \prod_{i=1}^5 X_i = 0 \right\}. \quad (58)$$

The complex structure modulus of the mirror-quintic is parametrized best by ζ (or z_ψ), where

$$\zeta := \frac{1}{2\pi i} \ln(z_\psi), \quad z_\psi := \frac{1}{(5\psi)^5}. \quad (59)$$

In such a Type IIB set-up, the two moduli fields (τ, ζ) correspond to what we loosely referred to as “complex structure moduli z ” in earlier sections. The period integral is expressed for a symplectic basis of $H_3(X; \mathbb{Z})$ as follows [22]

$$\Pi = \begin{pmatrix} \Pi_1 \\ \Pi_2 \\ \Pi_3 \\ \Pi_4 \end{pmatrix} = \begin{pmatrix} 1 \\ \zeta \\ -\frac{5}{2}\zeta^2 - \frac{11}{2}\zeta + \frac{25}{12} \\ \frac{5}{6}\zeta^3 + \frac{25}{12}\zeta - i\frac{\chi(X)\zeta_3}{(2\pi)^3} \end{pmatrix} + \mathcal{O}(e^{2\pi i n \zeta})_{n \geq 1}, \quad (60)$$

where ζ_3 is meant to be $\zeta(3) = 1.202 \dots$, and $\chi(X) = 200$ is the topological Euler number of the mirror-quintic Calabi–Yau 3-fold X . The Kähler potential of the chiral multiplet ζ is given by

$$\begin{aligned} K^{(\text{cpx})}/M_{\text{Pl}}^2 &= -\ln \left(i \int_X \Omega \wedge \overline{\Omega} \right) = -\ln (-i \Pi^\dagger \cdot \Sigma \cdot \Pi) \\ &= -\ln \left(\frac{5}{6} [(\zeta - \bar{\zeta})/i]^3 + \frac{50}{\pi^3} \zeta_3 + \mathcal{O}(e^{-2\pi \text{Im}(\zeta)}) \right) \end{aligned} \quad (61)$$

Changing the phase of the parameter z_ψ by 2π , we come back to the same point in the complex structure moduli space of the mirror-quintic. This results in a shift

$$\zeta \longrightarrow \zeta' = \zeta + 1, \quad (62)$$

but the Kähler potential above remains invariant. The period integral (60) does change under the shift (62), but the change is in the form of

$$\Pi(\zeta) \longrightarrow \Pi(\zeta + 1) = M_\infty \cdot \Pi(\zeta), \quad M_\infty := \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ -8 & -5 & 1 & 0 \\ 5 & -3 & -1 & 1 \end{pmatrix}, \quad (63)$$

and is regarded as monodromy transformation of a integral symplectic basis of $H_3(X; \mathbb{Z})$. This discrete shift symmetry may be regarded as a continuous shift symmetry (in the Kähler potential) approximately in the large complex structure region, $\text{Im}(\zeta) \gg 1$.

The complex structure for ζ and for $\zeta' = \zeta + 1$ are regarded the same physically, only in the absence of 3-form fluxes in Type IIB compactification. 3-form fluxes in Type IIB, $G = F^{(3)} - \tau H^{(3)}$, are characterized by the choice of integers $F^{(3)} \leftrightarrow \{n_a^R\}$ and $H^{(3)} \leftrightarrow \{n_a^{NS}\}$. For a given choice of fluxes, $\{n_a^R\}$ and $\{n_a^{NS}\}$, two choices of complex structure, ζ and ζ' are not equivalent in physics, because $\sum_a (n_a^R - \tau n_a^{NS}) \Pi_a(\zeta)$ is not the same as that for $\Pi_a(\zeta')$.

One could get the period integral $\Pi_a(\zeta'_a)$ back to $\Pi_a(\zeta)$ by the monodromy transformation, but the flux quanta in the new symplectic integral basis $\{n_a^R\}$ and $\{n_a^{NS}\}$ are not the same as before. Due to this monodromy mechanism on the moduli space of complex structure moduli, its covering space is a more appropriate moduli space for physics in the presence of flux on X .

It is possible that there are correction terms to the Kähler potential (61) generated in the presence of fluxes. However, corrections will be small, relatively by $\alpha' G^2 \sim 1/\langle\omega\rangle^3 \ll 1$ in the regime of our interest in this article ($\text{Re}\langle T\rangle \approx \mathcal{O}(10)$ or somewhat larger). We do not pay attention to the correction terms in the Kähler potential for this reason.

All the properties described so far are not specific to the mirror-quintic Calabi–Yau 3-fold, but hold true for most of Calabi–Yau 3-folds. “*All the properties*” include the approximate shift symmetry in the Kähler potential in the large complex structure moduli region in the absence of flux, the monodromy group action on the moduli space and the flux quanta, rational number coefficients in the period integral (like (60)) determined by topology of X as well as the mirror geometry of X [23], and the small corrections due to flux controlled by $1/[\text{Re}\langle T\rangle]^{3/2}$.

Such an idea of exploiting an approximate shift symmetry and monodromy in the complex structure moduli for inflation in Type IIB/F-theory has been presented in [6, 4, 5]; this is a mirror version of the same set of ideas exploited in axion monodromy inflation [24]. This set-up also shares with D3–D7 inflation an approximate continuous shift symmetry in the Kähler potential that is broken by flux or other effects in the superpotential [25]. In this sense, the use of complex structure moduli for string inflation is another variation of the same theme that has been pursued for the last decade.

5.1.2 Imaginary Part of ζ as an Inflaton?

In the large complex structure region $\text{Im}(\zeta) \gg 1$, the kinetic term is approximately

$$\mathcal{L} \simeq \frac{3M_{\text{Pl}}^2}{(\zeta - \bar{\zeta})^2} |\partial\zeta|^2. \quad (64)$$

This means that it is better to parametrize the complex ζ -plane by two real fields σ and φ as

$$\zeta = ie^{2\sigma}(1 + i2\varphi), \quad (65)$$

so that the kinetic term is close to the canonical one,

$$\mathcal{L} \simeq -3M_{\text{Pl}}^2 [(\partial\sigma)^2 + (\partial\varphi + 2(\partial\sigma)\varphi)^2]. \quad (66)$$

Because the superpotential W_{GVW} for the period integral in (60) is approximately a polynomial in ζ that is at most cubic, it is a combination of exponential functions in σ , and polynomial in φ . It is better to expect the slow-roll evolution along the φ direction, at least relatively to the σ direction. If we want the φ field to evolve by about $\mathcal{O}(10)$ – $\mathcal{O}(100)$ in order to earn sufficiently large e-fold number, then it means under the assumption of large complex structure region, $\sigma \gtrsim 1$, that the real part of ζ changes by of order $e^{2\sigma} \times (10\text{--}100)$.

5.1.3 A KYY Look-alike in Type IIB Orientifolds

If the real part of ζ were to change by $e^{2\sigma} \times \mathcal{O}(10) \gg 1$ in order to drive slow-roll inflation, the value of $\tilde{W}_{\text{eff}}^{(\text{cpx})}$ would also change by much more than a value of order unity. We have seen in section 4.2, however, that the scalar potential stabilizing the Kähler modulus T is distorted so much then that we are no longer confident whether T remains to be stabilized. It is thus possible to conclude that slow-roll inflation may be driven purely by the Kähler moduli field T , not by $z = (\zeta, \tau)$; right-handed sneutrino scenario may be relevant only through the kinetic/mass mixing between δT and δz around the supersymmetric minimum reached at the end of inflation, a possibility that we have already mentioned in section 2.

This constraint on the case for inflation in complex structure moduli has its origin in string compactification, because the limits on the variation in $W^{(\text{cpx})}$ came from stabilization of Kähler moduli. A similar limitation on large-field inflation model was encountered before, however, in the context of supergravity, rather than in string theory. That was an observation that it is not easy to implement chaotic inflation into 4D $\mathcal{N} = 1$ supergravity; starting with a simple monomial superpotential $W \sim \Phi^n$, we obtain a potential $|\Phi^{n-1}|^2$ from the F-term in rigid supersymmetry, but the $-3|W/M_{\text{Pl}}|^2$ term in the 4D supergravity scalar potential would become more important for super-Planckian Φ than the F-term. The potential not only stops being a monomial, but also even becomes negative. An idea to get around the problem in supergravity by Kawasaki, Yamaguchi and Yanagida (KYY) [26] was to consider an effective superpotential of the form $W = X \text{fcn}_1(\Phi)$ and a Kähler potential $K = X^\dagger X + \dots + \text{fcn}_2(\Phi + \Phi^\dagger)$ so that the value of X remains zero during inflation, and so do the values of W and $-3|W/M_{\text{Pl}}|^2$ consequently.

It appears that it is possible to implement such a situation by choosing the 3-form

fluxes.^{18,19} If we choose $n_1^R \neq 0$, $n_{2,3,4}^R = 0$ and n_a^{NS} 's generic in the mirror-quintic X , then

$$W^{(\text{cpx})} = \frac{M_{\text{Pl}}^3}{\sqrt{4\pi}} \left(n_1^R - \tau \left(\sum_a n_a^{NS} \Pi_a(\zeta) \right) \right). \quad (67)$$

Thus, if it is possible to choose n_1^R so that the tuning of Kallosh-Linde (6) is achieved,

$$\frac{M_{\text{Pl}}^3}{\sqrt{(4\pi)}} n_1^R + \langle W^{(T)} \rangle = 0, \quad (68)$$

then the remaining terms in the superpotential $W^{(\text{cpx})}(\zeta, \tau) - \langle W^{(\text{cpx})} \rangle$ is of the form introduced in [26]; τ is for X and $\sum_a n_a^{NS} \Pi_a(\zeta)$ for $\text{fcn}_1(\Phi)$. Can this combination remain very small during inflation, just like in [26], so that the problem of volume destabilization in string theory is also avoided?

The idea looks nice, but the tuning condition (68) can never be satisfied. The Ramond-Ramond 3-form flux quantum n_1^R is an integer, but the value of $\langle W^{(T)} \rangle$ is orders of magnitude smaller than M_{Pl}^3 as long as we work in the moderate large radius regime. If the Kallosh-Linde tuning (6) is to be achieved, that should be done not in a way as naive as in (68). Because of the value $\langle W^{(T)} \rangle \ll M_{\text{Pl}}^3/\sqrt{4\pi}$, the vacuum value $\langle W_{GVW} \rangle$ should vanish in some approximation; either small corrections in the approximation scheme or contributions to the superpotential $W^{(\text{tot})}$ other than W_{GVW} may achieve the tuning against $W^{(T)}$.

One can think of setting $n_1^R = 0$ and assume that there is a term other than $W^{(T)}$ and W_{GVW} in the superpotential $W^{(\text{tot})}$ whose vacuum value is W_0 . Such a contribution to $W^{(\text{tot})}$ may come, in principle, from condensation of operators in a gauge theory supported on D-branes in X . Thus, the tuning (6) may still be achieved. In this case, one should note that the Ramond-Ramond 3-form flux is completely absent. The F-term potentials of ζ and τ give rise to

$$V(T, \zeta, \tau) \supset \frac{1}{2\text{Im}(\tau)} \frac{1}{[2\text{Im}(\zeta)]^3} \frac{1}{[\text{Re}(T)]^3} \frac{M_{\text{Pl}}^4}{4\pi} \times |\tau|^2 \times \left\{ |f_N(\zeta)|^2 + 3|f_N(\zeta) - (1/3)(\zeta - \bar{\zeta})\partial_\zeta f_N(\zeta)|^2 \right\}, \quad (69)$$

driving $\text{Im}(\tau)$ to zero, similarly to the model of [26]. Here,

$$f_N(\zeta) := \sum_a n_a^{NS} \Pi_a(\zeta). \quad (70)$$

¹⁸See [27] for other attempts of implementing the idea of [26] in string theory.

¹⁹In the end, it turns out that this implementation does not work. Busy readers can proceed to section 5.2.

The current set-up is different from that in [26], however, in that the Kähler potential of τ is singular at $\tau = 0$ (remember that $K/M_{\text{Pl}}^2 \supset -\ln[(\tau - \bar{\tau})/i]$), whereas Ref. [26] assumes that the target space (X, Φ) is smooth at $X = 0$. The singularity of the Kähler potential at $\tau = 0$ and a possible ζ -dependence of the coefficients a_i 's in $W^{(T)}$ results in such terms as

$$V(T, \zeta, \tau) \supset \frac{1}{\text{Im}(\tau)} \frac{|\partial_\zeta W^{(T)}|^2}{M_{\text{Pl}}^2}. \quad (71)$$

Eventually a minimum may be formed around $\text{Im}(\tau) \sim W^{(T)}/[M_{\text{Pl}}^3 f_N(\zeta)]$. If all things work properly in this way, and if the field value of τ keeps track of this minimum while the value of ζ changes over time, then the combination $M_{\text{Pl}}^3 \tau f_N(\zeta) \sim W_{GVW}$ remains of order $\langle W^{(T)} \rangle$. This observation brings a hope that the volume stabilization constraint in section 4.2 may actually be satisfied.

The crude argument above does not pay close enough attention to the power counting of $\text{Im}(\zeta)$ or $\text{Re}(T)$, whose value we assume to be somewhat larger than 1. Given the fact that the constraints on $W^{(\text{cpx})}$ —both the upper bound and lower bound—come at the same order as its vacuum value $W_0 = -\langle W^{(T)} \rangle$, the power counting of $\text{Re}(T)$ and $\text{Im}(\zeta)$ should be a crucial step to see whether the Type IIB implementation [(67) with $n_1^R = 0$] of [26] works or not. We did not choose to carry out such a careful study because we are referring to a potential $V(T, \zeta, \tau)$ in the $\text{Im}(\tau) \ll 1$ region, and small $\text{Im}(\tau) = e^{-\phi}$ implies large string coupling. The Kähler potential will receive corrections to the one $K \propto -\ln[(\tau - \bar{\tau})/i]$ in the strong coupling $e^\phi \gg 1$ regime, and it is hard to see to what extent such an analysis in perturbative Type IIB string theory is reliable.

If we are to seek for a set up that is $\text{SL}(2; \mathbb{Z})$ -equivalent to the flux superpotential (67) with $n_1^R = 0$, that is in the form of

$$W_{GVW} = M_{\text{Pl}}^3 \frac{1}{\sqrt{4\pi}} (n^R - \tau n^{NS}) \cdot f(\zeta), \quad (72)$$

with n^R and n^{NS} both integers. Repeating the same analysis as above, however, we see that the valley in the scalar potential $V(\zeta, \tau, T)$ is in

$$n^R - \tau n^{NS} \approx \frac{W^{(T)} \sqrt{4\pi} / M_{\text{Pl}}^3}{\partial_\zeta f(\zeta)}; \quad (73)$$

the combination $(n^R - \tau n^{NS})$ remains small along the potential valley, as in [26]; the problem is that $\text{Im}(\tau)$ also remains very small along the valley, and the string coupling large, provided $n^{NS} \neq 0$. Thus, such an $\text{SL}(2; \mathbb{Z})$ -equivalent description is still unreliable.

The only $\text{SL}(2; \mathbb{Z})$ -equivalent description of (67) with $e^\phi \ll 1$ is the one with $n^{NS} = 0$. This time, only the Ramond–Ramond fluxes are present, and the NS–NS flux is completely absent. This then suggests that the dilaton cannot be stabilized by the Gukov–Vafa–Witten superpotential. Indeed, one can see by writing down the scalar potential of the total system that the potential has a runaway direction $\text{Im}(\tau) \rightarrow \infty$; although the total system includes additional terms (such as gaugino condensations) other than W_{GVW} , it is sufficient for justification of the runaway-claim above to assume that the Kallosh–Linde tuning (6) is achieved in a stabilized vacuum.

If there were a stabilized minimum reached after inflation in the description (67) with $n_1^R = 0$ or (72), then it should be possible to map the vacuum into the weak string coupling region by the $\text{SL}(2; \mathbb{Z})$ transformation. That fact that we can find the runaway situation at best implies that there is no stable minimum in the two earlier descriptions, either. The hope is dashed, and we abandon for now the idea of using the $(n^R - \tau n^{NS}) \cdot f(\zeta)$ production structure in Type IIB Calabi–Yau orientifolds, until we will recycle this idea in section 5.3.

5.2 Deformation around Approximately Arithmetic Complex Structure

It is easy to imagine that the constraint in section 4.2 is so tight without an idea like the one we pursued in section 5.1, that only very negligible e-fold is achieved. This statement is so obvious intuitively that we do not think it is necessary to build up precise estimate of the e-fold. The constraint in section 4.2 was derived, however, by using the potential $V_{\text{racetrack}}(T)$ in combination with guess work. The technical presentation in the following is the first step one needs to take in order to verify or falsify the “guess work” part of the discussion in section 4.2. We also believe that the following discussion (making an estimate of e-fold) will remain useful also when some ideas (like those in section 5.3) are implemented.

Obviously we need to take on both of these problems:

- (a) find a vacuum in string theory where the Kallosh–Linde tuning (6) is achieved at least approximately at the potential minimum $(T, z) = (\langle T \rangle, \langle z \rangle)$, and
- (b) make sure that the volume destabilization constraint is satisfied during the inflation / reheating process.

A lesson from the study in section 5.1.3 is that the issue (a) itself is not an easy problem, because of the quantization condition of the flux and hierarchically small value we expect for $\langle W^{(T)} \rangle$.

Let us study the issue (a) a little more systematically than in section 5.1.3. We assume that the vacuum is found in the large complex structure $\text{Im}(\zeta) \gtrsim 1$ region,²⁰ and employ a presentation where there is only one complex structure modulus field ζ (as in the mirror quintic); generalization to cases with $h^{2,1}(X) > 1$ may be possible, but we will not discuss.

We consider it is an important clue in thinking about the issue (a) that the vacuum value $\langle W^{(T)} \rangle$ is hierarchically small relatively to $M_{\text{Pl}}^3/\sqrt{4\pi}$ in the moderately large radius regime ($\text{Re}(\langle T \rangle) \approx \mathcal{O}(10)$ or a little more); the factor $1 \gg 1/[(4\pi)(\text{Re} \langle T \rangle)^{3/2}]$ and additional exponential factors are unavoidable. Compared against this is the combination $\sum_a (n_a^R - n_a^{NS} \tau) \Pi_a$, which does not contain any small value. Thus, the tuning condition of Kallosh–Linde (6) is understood primarily as $\langle W_{GVW} \rangle \approx 0$ in the sense that $|\langle W_{GVW} \rangle|$ is much smaller than $M_{\text{Pl}}^3 \times \mathcal{O}(1)$. Minimum finding problem of the superpotential W_{GVW} with the condition $\langle W_{GVW} \rangle \approx 0$ has been addressed in the literature such as [28].

In the large complex structure region $\text{Im}(\zeta) \gtrsim 1$ of the moduli space, the period integral (60) is already given in the form of power-series expansion in $e^{2\pi i \zeta}$ (world-sheet instanton expansion in the mirror of X), and we can think²¹ of using this approximation scheme in order to give more precise meaning in the statement $\langle W_{GVW} \rangle \simeq 0$ above.

To be more explicit, we drop all the $\mathcal{O}(e^{2\pi i n \zeta})_{n \geq 1}$ parts and also the $i\zeta_3$ term from period integrals (60) to define $\Pi_a^{\text{poly}}(\zeta)$, and

$$f_R^{\text{poly}}(\zeta) := \sum_a n_a^R \Pi_a^{\text{poly}}(\zeta), \quad f_N^{\text{poly}}(\zeta) := \sum_a n_a^{NS} \Pi_a^{\text{poly}}(\zeta). \quad (74)$$

At this level of approximation, we have $W_{GVW}^{\text{poly}} = M_{\text{Pl}}^3/\sqrt{4\pi} \times (f_R^{\text{poly}}(\zeta) - \tau f_N^{\text{poly}}(\zeta))$. The argument above implies that we should require

$$[W_{GVW}^{\text{poly}}] = 0 \quad (75)$$

at the approximate vacuum value $(\tau, \zeta) = (\tau_*, \zeta_*)$ characterized by

$$\partial_\tau [W_{GVW}^{\text{poly}}] = 0, \quad \partial_\zeta [W_{GVW}^{\text{poly}}] = 0. \quad (76)$$

This means that

$$f_N^{\text{poly}}(\zeta_*) = 0, \quad \tau_* = \frac{\partial_\zeta f_R^{\text{poly}}(\zeta_*)}{\partial_\zeta f_N^{\text{poly}}(\zeta_*)}, \quad f_R^{\text{poly}}(\zeta_*) = 0. \quad (77)$$

²⁰ It is logically possible that inflation takes place in the $\text{Im}(\zeta) \gg 1$ region, but the vacuum is in the region $\text{Im}(\langle \zeta \rangle) < 1$. Such a case is not covered in this section 5.2.

²¹ See footnote 22.

With the definition of f_N^{poly} and f_R^{poly} introduced above, we can still maintain connection between arithmetics and flux vacua over the entire region of $\text{Im}(\zeta) \gtrsim 1$, although the connection is now under an approximation scheme (see [29]).²² In Calabi–Yau orientifolds of Type IIB string, $f_{N,R}^{poly}$ are always at most cubic polynomial of ζ , and the coefficients take values in \mathbb{Q} . The approximate vacuum value (ζ_*, τ_*) satisfying (77) are always algebraic numbers, and they generate an algebraic number field $\mathbb{Q}[\zeta_*, \tau_*]$, once all the flux quanta are fixed.

There are two distinct cases whose consequences are quite different. One is the case all the three roots of f_R^{poly} are the same as those of f_N^{poly} . There is a common $f^{poly}(\zeta)$ so that $f_R^{poly} = n_4^R f^{poly}(\zeta)$ and $f_N^{poly} = n_4^{NS} f^{poly}(\zeta)$, and we go back to the case we have already seen in (72). All the other cases have the property that

$$\dim_{\mathbb{Q}}(\mathbb{Q}[\zeta_*, \tau_*]) = 2; \quad (78)$$

to see this, it is enough to see that ζ_* is a root of a quadratic polynomial

$$f_N^{poly}(\zeta)/n_4^{NS} - f_R^{poly}(\zeta)/n_4^R \neq 0 \quad (79)$$

with all the coefficients being rational.

In the latter case, the two polynomials can be written down as

$$f_R^{poly}(\zeta) = -\frac{5}{6} n_4^R (\zeta - \zeta_*)(\zeta - \bar{\zeta}_*)(\zeta - \zeta_R), \quad (80)$$

$$f_N^{poly}(\zeta) = -\frac{5}{6} n_4^{NS} (\zeta - \zeta_*)(\zeta - \bar{\zeta}_*)(\zeta - \zeta_N), \quad (81)$$

with ζ_R and ζ_N in \mathbb{Q} , and $\zeta_N \neq \zeta_R$. It follows that

$$\tau_* = \frac{n_4^R}{n_4^{NS}} \frac{\zeta_* - \zeta_R}{\zeta_* - \zeta_N}. \quad (82)$$

If $n_4^R \gg n_4^{NS}$, the vacuum value of dilation is in the weak coupling region, $\text{Im}(\tau_*) \gg 1$, and the existence of such a vacuum is reliable.

²² The argument here is only to present an idea of how to achieve the Kallosh–Linde tuning, and we do not mean to say this is the only possibility. Indeed, the vacuum value $\langle W_{GVW} \rangle$ needs to be zero only approximately (relatively to $M_{\text{Pl}}^3 \times \mathcal{O}(1)$). The relation (75) or $f_R^{poly}(\zeta_*) = 0$ needs to hold only approximately. An alternative to the idea we adopted in the main text is that $\Pi_a(\zeta)$'s and $\partial_\zeta \Pi_a(\zeta)$'s take their values in some algebraic extension field $K(\zeta)$ and somehow $f_N(\zeta) = f_R(\zeta) = 0$ —(★), rather than $\Pi_a^{poly}(\zeta)$, $\partial_\zeta \Pi_a^{poly}(\zeta)$ do [29, 30]. In this alternative, $f_N^{poly} = 0$ and $f_R^{poly} = 0$ may not have a common solution, though ζ_* for $f_N^{poly}(\zeta_*) = 0$ should be very close to one of the solutions to $f_R^{poly} = 0$. The true vacuum value of ζ will be shifted from the one satisfying (★), because of the ζ dependence of the prefactors a_i in the gaugino condensation terms. This shift in $\langle \zeta \rangle$, and hence that in W_{GVW} will be exponentially small, so that there is still a hope that there is some non-trivial cancellation mechanism between this shift and the value $\langle W^{(T)} \rangle$.

All the argument, especially with the vacuum value (ζ_*, τ_*) satisfying (77), does not guarantee that $\langle W_{GVW} \rangle / M_{\text{Pl}}^3 \ll \mathcal{O}(1)$, however. This is because we have set aside the $i\chi(X)\zeta_3$ terms from Π_a^{poly} 's, so that we can maintain the connection with arithmetic vacuum value of the complex structure moduli/period integrals. In the presence of these terms in the period integral Π_a , in fact, it follows that $\langle W_{GVW} \rangle \gg M_{\text{Pl}}^3$. To see this, let us first define $\tilde{f}_R^{\text{poly}}(\zeta)$ and $\tilde{f}_N^{\text{poly}}(\zeta)$, just like $f_R^{\text{poly}}(\zeta)$ and $f_N^{\text{poly}}(\zeta)$, but without dropping the term $i\zeta_3 \times \chi(X)/(2\pi)^3$ from the period integral in (60). The solution ζ_* of $f_N^{\text{poly}}(\zeta) = 0$ is slightly shifted to be $\tilde{\zeta}_*$ for $\tilde{f}_N^{\text{poly}}(\tilde{\zeta}_*) = 0$. The shift $\tilde{\zeta}_* - \zeta_*$ remains very small, provided ζ_* is in the $\text{Im}\zeta_* \gg 1$ region and the approximation scheme works well. For this small shift, however, the (approximate) vacuum value of

$$(\tilde{f}_R^{\text{poly}}(\zeta) - \tau \tilde{f}_N^{\text{poly}}(\zeta))|_{\zeta=\tilde{\zeta}_*} = -\frac{5}{6}n_4^R(\zeta_N - \zeta_R)(\zeta - \zeta_*)(\zeta - \bar{\zeta}_*)|_{\zeta=\tilde{\zeta}_*}, \quad (83)$$

is evaluated to be approximately

$$-n_4^{NS}\text{Im}(\tau_*) \times i n_4^{NS} \frac{\chi(X)\zeta_3}{(2\pi)^3}; \quad (84)$$

this means that we need to give up either $\text{Im}(\tau_*) \gg 1$ or $\langle W_{GVW} \rangle \ll M_{\text{Pl}}^3$.

The discussion above also reveals that there is an obvious loop hole. Calabi–Yau 3-folds with $\chi(X) = 0$ have a special properties that the $i\zeta_3/(2\pi)^3$ term drops out from the period integral (see [29] however). In this case, $\tilde{f}_N^{\text{poly}} = f_N^{\text{poly}}$ and $\tilde{f}_R^{\text{poly}} = f_R^{\text{poly}}$, so that $\zeta = \zeta_*$ remains to be the solution of both. The Kallosh–Linde tuning condition is maintained at least at the level of terms of order $M_{\text{Pl}}^3 \times \mathcal{O}(1)$; cancellation involving the “mirror-worldsheet-instanton” correction terms $\mathcal{O}(e^{2\pi i n \zeta})$'s and $\langle W^{(T)} \rangle$ remains to be an open problem, however. The string coupling can be small at the vacuum, because

$$\text{Im}(\tau_*) = \frac{n_4^R}{n_4^{NS}} \text{Im} \left[\frac{\zeta_N - \zeta_R}{\zeta_* - \zeta_N} \right] \quad (85)$$

can be made much larger than unity by taking (n_4^R/n_4^{NS}) sufficiently large. Unlike in the cases we studied in section 5.1.3, we have a vacuum in the reliable weak coupling regime. This holds as an approximate solution²³ to the issue (a).

With a vacuum of complex structure formulated, it is now possible to discuss how much the complex structure can be deformed from the vacuum value during the slow-roll inflation/reheating process, i.e., the issue (b). Since we have imposed conditions that $W_{GVW} \approx 0$

²³To be rigorous, one should also ask whether there is a D7-brane configuration for the $SU(N_1) \times SU(N_2)$ gaugino condensation satisfying the calibration condition in the presence of fluxes. F-theory will be a better tool to study this issue, however.

and $\partial_\zeta W_{GVW} \approx 0$ at $\zeta \approx \zeta_*$, the value in $\widetilde{\delta W}_{\text{eff}}^{(\text{cpx})}$ can be evaluated by using

$$\frac{\partial^2}{\partial \zeta^2} \left[f_R^{\text{poly}}(\zeta) - \tau_* f_N^{\text{poly}}(\zeta) \right] \Big|_{\zeta=\zeta_*} = -\frac{5}{3} n_4^R (\zeta_* - \bar{\zeta}_*) \frac{\zeta_R - \zeta_N}{\zeta_* - \zeta_N} \sim \text{Im}(\tau_*) \times n_4^{NS} (\zeta_* - \bar{\zeta}_*). \quad (86)$$

We have chosen parameters, so that this second derivative is larger than $\mathcal{O}(1)$; $\text{Im}(\zeta_*) \gg 1$ so that the approximation of dropping the mirror world-sheet instanton terms are small, and the string coupling is also small ($\text{Im}(\tau_*) \gg 1$). The constraint (54) therefore implies that the complex structure deformation $\delta\zeta = \zeta - \zeta_*$ is allowed without volume destabilization, at most only at the level of $\sqrt{10^{-3}/\text{Im}(\zeta)}$. This is much smaller than the variation $\delta\zeta \sim \text{Im}(\zeta) \times \mathcal{O}(10 - 100)$ we need for sufficient e-fold.

5.3 Possible Solutions of Inflation

We have seen so far that it is not easy at all for complex structure moduli field to play some role in inflation. If we still try to seek for the “right-handed sneutrino scenario” to be relevant somehow in the Type IIB/F-theory compactifications with moderately large internal volume, that will have to be in a very special situation. The discussion in this article helps us pin down such possibilities. Let us describe a few of these at the end of this article.

I: The first possibility is still to try to seek for an analogue of [26] or (67), so that $(\delta\zeta)^2 \times (\partial_\zeta^2 W_{GVW})|_{\zeta=\zeta_*}$ remains small or vanishes during inflation. At least two fields are necessary then, and some structure needs to be present in the space of complex structure moduli. Although we have seen in section 5.1.3 that the flux superpotential for Type IIB orientifolds (67) does not work, we can think of F-theory compactification over a Calabi–Yau 4-fold $Y_4 = \text{K3} \times \text{K3} = S_1 \times S_2$, where the period integral is in a factorized form $\Omega_Y = \Omega_{S_1} \cdot \Omega_{S_2}$. This is an analogue of (67) in Type IIB orientifolds, in that the factorized form of $(n^R - \tau n^{NS}) \cdot \Pi$ originates from the product structure in $Y_4 = (T^2 \times X)/\mathbb{Z}_2$. This F-theory version also has advantages of being able to handle large g_s region of the moduli space, and of being able to generate up-type Yukawa couplings in supersymmetric unified theories through its E_6 algebra [31]. It is known in the $Y_4 = \text{K3} \times \text{K3}$ compactification of F-theory that some type of flux [a part of what is called “ G_0 -type” in [32]] preserves the factorized structure of the period integral and gives rise to pure Dirac type mass matrix in the superpotential [33].

A down side of the $\text{K3} \times \text{K3}$ compactification of F-theory is that all the 7-branes are parallel, so that quarks and leptons are not generated in 7-brane intersections. Thus, instead of taking a Calabi–Yau 4-fold to be purely in a direct product, $Y_4 = S_1 \times S_2$, one may think

of Y_4 being a K3-fibration²⁴ over a complex surface S , and hope that something similar to the product structure of the period integral $\Pi_{S_1} \times \Pi_{S_2}$ still remains even in the fibred geometry, at least approximately somewhere in the moduli space. The authors do not know whether that is true. References [37, 38] contain useful information in extending the study of section 5.2 in such F-theory compactifications.²⁵

II: As a second possibility, one can think of topological inflation, if the W_{GVW} superpotential for a given flux admits multiple minima. One minimum may be in the large complex structure region, $\text{Im}\zeta \gtrsim 1$, though the other minimum may not be. It is a rule of thumb that topological inflation takes place if the distance (measured in the metric of the non-linear sigma model) between the minima exceeds M_{Pl} . At least by choosing flux quanta properly,²⁶ one of the minima can be moved to large $\text{Im}\zeta$ region. In the middle of a domain wall in between the two minima, the volume stabilization constraint discussed in section 4 may no longer be satisfied, however. The authors are not sure whether such a field configuration leads to topological inflation, and whether the predicted fluctuations in CMB are consistent with all the observations.

III: One cannot rule out logically that the complex structure ζ dependence of the prefactors a_i in the racetrack superpotential is such that the barrier in the volume stabilization potential remains high enough under the evolution of ζ during inflation (c.f. [39]).

IV: The bottom-up idea of “right-handed sneutrino inflation” scenario has two ingredients: i) the scalar partner of the right-handed neutrinos and the Majorana mass provides the potential of chaotic inflation, and ii) the reheating process proceeds dominantly through the renormalizable interactions of right-handed neutrinos. Even when inflation is driven purely by the Kähler moduli field T , the reheating process may proceed through right-handed neutrinos (which are part of complex structure moduli [2]), due to the mixing between the Kähler

²⁴It follows in this case [34] that an elegant mechanism of $\text{SU}(5)_{\text{GUT}} \rightarrow \text{SU}(3)_C \times \text{SU}(2)_L \times \text{U}(1)_Y$ symmetry breaking in [35, 34] cannot be used. There are other mechanism for this symmetry breaking, however; see [34, 36] for more information.

²⁵The analogue of $\chi(Z) = 0$ Calabi–Yau 3-folds in section 5.2 will be a 4-fold Y_4 whose mirror \tilde{Y}_4 has vanishing third Chern class.

²⁶We can think of flux of the following property: $n_4^{NS} \approx \mathcal{O}(1)$, $n_3^{NS} \approx \mathcal{O}(K)$, $n_2^{NS} \approx \mathcal{O}(K^2)$ and $n_1^{NS} \approx \mathcal{O}(K^3)$, where K is a number much larger than 1. When the NS-NS 3-form flux is chosen in that way, all of the three solutions to $f_N^{\text{poly}}(\zeta) = 0$ satisfy $|\zeta| \approx \mathcal{O}(K)$, one of them has an $\mathcal{O}(K)$ positive imaginary part, another has an $\mathcal{O}(K)$ negative imaginary part, and the last one, ζ_N , being real. Thus, one of the three solutions, (and in fact only one of them), denoted by ζ_* , is in the region where $\text{Im}\zeta \gg 1$. Similar argument holds also for the roots of $f_R^{\text{poly}} = 0$. It should be remembered that we cannot take this scaling parameter K to be arbitrarily large, however. There is an upper bound in the flux contribution to the D3-tadpole. $\text{Im}(\zeta_*)$ should not be too large either, because winding stringy states will be in the light spectrum [40].

moduli and complex structure moduli fields, as we discussed in section 2. Therefore, the volume destabilization limit on the complex structure deformation can be avoided, but the aspect ii) of the right-handed sneutrino inflation scenario is still realized in this case. Various predictions on the thermal history after inflation will be lost, however, because the reheating process depends on various details of spectrum and mixing of Kähler and complex structure moduli at the vacuum, not just on the physics of right-handed neutrinos.

Acknowledgements

We thank T.T.Yanagida for stimulating discussion, and for giving us his comments on our manuscript. HH would like to thank Mainz Institute for Theoretical Physics for hospitality and its partial support during a part of this work. This work is supported by the REA grant agreement PCIG10-GA-2011-304023 from the People Programme of FP7 (Marie Curie Action) (H.H.), the grant FPA2012-32828 from the MINECO (H.H.), the ERC Advanced Grant SPLE under contract ERC-2012-ADG-20120216-320421 (H.H.), the grant SEV-2012-0249 of the “Centro de Excelencia Severo Ochoa” Programme (H.H.), WPI Initiative (R.M., T.W.), Advanced Leading Graduate Course for Photon Science grant (R.M.), and a Grant-in-Aid for Scientific Research on Innovative Areas 2303, MEXT, Japan (T.W.).

References

- [1] H. Murayama, H. Suzuki, T. Yanagida and J. Yokoyama, “Chaotic inflation and baryogenesis in supergravity,” *Phys. Rev. D* **50**, 2356 (1994) [hep-ph/9311326];
K. Hamaguchi, H. Murayama and T. Yanagida, “Leptogenesis from N dominated early universe,” *Phys. Rev. D* **65**, 043512 (2002) [hep-ph/0109030];
H. Murayama, K. Nakayama, F. Takahashi and T. T. Yanagida, “Sneutrino Chaotic Inflation and Landscape,” arXiv:1404.3857 [hep-ph].
- [2] R. Tatar, Y. Tsuchiya and T. Watari, “Right-handed Neutrinos in F-theory Compactifications,” *Nucl. Phys. B* **823**, 1 (2009) [arXiv:0905.2289 [hep-th]].
- [3] E. Witten, “New Issues in Manifolds of SU(3) Holonomy,” *Nucl. Phys. B* **268**, 79 (1986).
- [4] L. McAllister, E. Silverstein, A. Westphal and T. Wrase, “The Powers of Monodromy,” *JHEP* **1409**, 123 (2014) [arXiv:1405.3652 [hep-th]].
- [5] A. Hebecker, S. C. Kraus and L. T. Witkowski, “D7-Brane Chaotic Inflation,” *Phys. Lett. B* **737**, 16 (2014) [arXiv:1404.3711 [hep-th]]; M. Arends, A. Hebecker, K. Heimpel,

- S. C. Kraus, D. Lust, C. Mayrhofer, C. Schick and T. Weigand, “D7-Brane Moduli Space in Axion Monodromy and Fluxbrane Inflation,” *Fortsch. Phys.* **62**, 647 (2014) [arXiv:1405.0283 [hep-th]].
- [6] R. Blumenhagen and E. Plauschinn, “Towards Universal Axion Inflation and Reheating in String Theory,” *Phys. Lett. B* **736**, 482 (2014) [arXiv:1404.3542 [hep-th]]. R. Blumenhagen, D. Herschmann and E. Plauschinn, “The Challenge of Realizing F-term Axion Monodromy Inflation in String Theory,” arXiv:1409.7075 [hep-th].
- [7] E. Palti and T. Weigand, “Towards large r from $[p, q]$ -inflation,” *JHEP* **1404**, 155 (2014) [arXiv:1403.7507 [hep-th]]. F. Marchesano, G. Shiu and A. M. Uranga, “F-term Axion Monodromy Inflation,” arXiv:1404.3040 [hep-th]. T. W. Grimm, “Axion Inflation in F-theory,” arXiv:1404.4268 [hep-th]. L. E. Ibañez and I. Valenzuela, “The inflaton as an MSSM Higgs and open string modulus monodromy inflation,” *Phys. Lett. B* **736**, 226 (2014) [arXiv:1404.5235 [hep-th]]. R. Kappl, S. Krippendorf and H. P. Nilles, “Aligned Natural Inflation: Monodromies of two Axions,” *Phys. Lett. B* **737**, 124 (2014) [arXiv:1404.7127 [hep-th]]. C. Long, L. McAllister and P. McGuirk, “Aligned Natural Inflation in String Theory,” *Phys. Rev. D* **90**, 023501 (2014) [arXiv:1404.7852 [hep-th]]. S. Franco, D. Galloni, A. Retolaza and A. Uranga, “Axion Monodromy Inflation on Warped Throats,” arXiv:1405.7044 [hep-th]. X. Gao, T. Li and P. Shukla, *JCAP* **1410**, no. 10, 048 (2014) [arXiv:1406.0341 [hep-th]]. I. Ben-Dayan, F. G. Pedro and A. Westphal, “Towards Natural Inflation in String Theory,” arXiv:1407.2562 [hep-th]. Z. Kenton and S. Thomas, “D-brane Potentials in the Warped Resolved Conifold and Natural Inflation,” arXiv:1409.1221 [hep-th].
- [8] S. Gukov, C. Vafa and E. Witten, “CFT’s from Calabi-Yau four folds,” *Nucl. Phys. B* **584**, 69 (2000) [Erratum-ibid. *B* **608**, 477 (2001)] [hep-th/9906070].
- [9] S. Kachru, M. B. Schulz and S. Trivedi, “Moduli stabilization from fluxes in a simple IIB orientifold,” *JHEP* **0310**, 007 (2003) [hep-th/0201028].
- [10] R. Kallosh and A. D. Linde, “Landscape, the scale of SUSY breaking, and inflation,” *JHEP* **0412**, 004 (2004) [hep-th/0411011].
- [11] S. B. Giddings, S. Kachru and J. Polchinski, “Hierarchies from fluxes in string compactifications,” *Phys. Rev. D* **66**, 106006 (2002) [hep-th/0105097].
- [12] S. Kachru, R. Kallosh, A. D. Linde, J. M. Maldacena, L. P. McAllister and S. P. Trivedi, “Towards inflation in string theory,” *JCAP* **0310**, 013 (2003) [hep-th/0308055].

- [13] W. Buchmuller, K. Hamaguchi, O. Lebedev and M. Ratz, “Dilaton destabilization at high temperature,” Nucl. Phys. B **699**, 292 (2004) [hep-th/0404168].
- [14] H. Jockers and J. Louis, “The Effective action of D7-branes in $N = 1$ Calabi-Yau orientifolds,” Nucl. Phys. B **705**, 167 (2005) [hep-th/0409098]; H. Jockers and J. Louis, “D-terms and F-terms from D7-brane fluxes,” Nucl. Phys. B **718**, 203 (2005) [hep-th/0502059].
- [15] N. V. Krasnikov, “On Supersymmetry Breaking in Superstring Theories,” Phys. Lett. B **193**, 37 (1987); J. A. Casas, Z. Lalak, C. Munoz and G. G. Ross, “Hierarchical Supersymmetry Breaking and Dynamical Determination of Compactification Parameters by Nonperturbative Effects,” Nucl. Phys. B **347**, 243 (1990); T. R. Taylor, “Dilaton, gaugino condensation and supersymmetry breaking,” Phys. Lett. B **252**, 59 (1990).
- [16] G. D. Coughlan, W. Fischler, E. W. Kolb, S. Raby and G. G. Ross, “Cosmological Problems for the Polonyi Potential,” Phys. Lett. B **131**, 59 (1983). J. R. Ellis, D. V. Nanopoulos and M. Quiros, “On the Axion, Dilaton, Polonyi, Gravitino and Shadow Matter Problems in Supergravity and Superstring Models,” Phys. Lett. B **174**, 176 (1986). B. de Carlos, J. A. Casas, F. Quevedo and E. Roulet, “Model independent properties and cosmological implications of the dilaton and moduli sectors of 4-d strings,” Phys. Lett. B **318**, 447 (1993) [hep-ph/9308325]. T. Banks, D. B. Kaplan and A. E. Nelson, “Cosmological implications of dynamical supersymmetry breaking,” Phys. Rev. D **49**, 779 (1994) [hep-ph/9308292]. L. Randall and S. D. Thomas, “Solving the cosmological moduli problem with weak scale inflation,” Nucl. Phys. B **449**, 229 (1995) [hep-ph/9407248]. T. Banks, M. Berkooz and P. J. Steinhardt, “The Cosmological moduli problem, supersymmetry breaking, and stability in postinflationary cosmology,” Phys. Rev. D **52**, 705 (1995) [hep-th/9501053]. M. Dine, L. Randall and S. D. Thomas, “Supersymmetry breaking in the early universe,” Phys. Rev. Lett. **75**, 398 (1995) [hep-ph/9503303].
- [17] F. Denef, M. R. Douglas, B. Florea, A. Grassi and S. Kachru, “Fixing all moduli in a simple f-theory compactification,” Adv. Theor. Math. Phys. **9**, 861 (2005) [hep-th/0503124].
- [18] M. Dine, R. Rohm, N. Seiberg and E. Witten, “Gluino Condensation in Superstring Models,” Phys. Lett. B **156**, 55 (1985).
- [19] J. Wess and J. Bagger, “Supersymmetry and Supergravity (2nd ed.),” Princeton University Press (1992).
- [20] V. Kaplunovsky and J. Louis, “Field dependent gauge couplings in locally supersymmetric effective quantum field theories,” Nucl. Phys. B **422**, 57 (1994) [hep-th/9402005].

- [21] C. Escoda, M. Gomez-Reino and F. Quevedo, “Saltatory de Sitter string vacua,” JHEP **0311**, 065 (2003) [hep-th/0307160].
- [22] P. Candelas, X. C. De La Ossa, P. S. Green and L. Parkes, “A Pair of Calabi-Yau manifolds as an exactly soluble superconformal theory,” Nucl. Phys. B **359**, 21 (1991).
- [23] S. Hosono, A. Klemm, S. Theisen and S. T. Yau, “Mirror symmetry, mirror map and applications to complete intersection Calabi-Yau spaces,” Nucl. Phys. B **433**, 501 (1995) [hep-th/9406055].
- [24] E. Silverstein and A. Westphal, “Monodromy in the CMB: Gravity Waves and String Inflation,” Phys. Rev. D **78**, 106003 (2008) [arXiv:0803.3085 [hep-th]]. L. McAllister, E. Silverstein and A. Westphal, “Gravity Waves and Linear Inflation from Axion Monodromy,” Phys. Rev. D **82**, 046003 (2010) [arXiv:0808.0706 [hep-th]].
- [25] C. Herdeiro, S. Hirano and R. Kallosh, “String theory and hybrid inflation / acceleration,” JHEP **0112**, 027 (2001) [hep-th/0110271]. K. Dasgupta, C. Herdeiro, S. Hirano and R. Kallosh, “D3 / D7 inflationary model and M theory,” Phys. Rev. D **65**, 126002 (2002) [hep-th/0203019]. J. P. Hsu, R. Kallosh and S. Prokushkin, “On brane inflation with volume stabilization,” JCAP **0312**, 009 (2003) [hep-th/0311077]. F. Koyama, Y. Tachikawa and T. Watari, “Supergravity analysis of hybrid inflation model from D3 - D7 system,” Phys. Rev. D **69**, 106001 (2004) [Erratum-ibid. D **70**, 129907 (2004)] [hep-th/0311191]. J. P. Hsu and R. Kallosh, “Volume stabilization and the origin of the inflaton shift symmetry in string theory,” JHEP **0404**, 042 (2004) [hep-th/0402047]. M. Berg, M. Haack and B. Kors, “Loop corrections to volume moduli and inflation in string theory,” Phys. Rev. D **71**, 026005 (2005) [hep-th/0404087].
- [26] M. Kawasaki, M. Yamaguchi and T. Yanagida, “Natural chaotic inflation in supergravity,” Phys. Rev. Lett. **85**, 3572 (2000) [hep-ph/0004243].
- [27] E. Dudas, “Three-form multiplet and Inflation,” arXiv:1407.5688 [hep-th].
- [28] A. Giriyavets, S. Kachru, P. K. Tripathy and S. P. Trivedi, “Flux compactifications on Calabi-Yau threefolds,” JHEP **0404**, 003 (2004) [hep-th/0312104]. O. DeWolfe, A. Giriyavets, S. Kachru and W. Taylor, “Enumerating flux vacua with enhanced symmetries,” JHEP **0502**, 037 (2005) [hep-th/0411061]. J. J. Blanco-Pillado, R. Kallosh and A. D. Linde, “Supersymmetry and stability of flux vacua,” JHEP **0605**, 053 (2006) [hep-th/0511042]. N. C. Bizet, A. Klemm and D. V. Lopes, “Landscaping with fluxes and the E8 Yukawa Point in F-theory,” arXiv:1404.7645 [hep-th].

- [29] G. W. Moore, “Attractors and arithmetic,” hep-th/9807056. G. W. Moore, “Arithmetic and attractors,” hep-th/9807087. G. W. Moore, “Les Houches lectures on strings and arithmetic,” hep-th/0401049.
- [30] DeWolfe et. al. in [28].
- [31] R. Tatar and T. Watari, “Proton decay, Yukawa couplings and underlying gauge symmetry in string theory,” Nucl. Phys. B **747**, 212 (2006) [hep-th/0602238]. R. Donagi and M. Wijnholt, “Model Building with F-Theory,” Adv. Theor. Math. Phys. **15**, 1237 (2011) [arXiv:0802.2969 [hep-th]]. C. Beasley, J. J. Heckman and C. Vafa, “GUTs and Exceptional Branes in F-theory - I,” JHEP **0901**, 058 (2009) [arXiv:0802.3391 [hep-th]]. H. Hayashi, R. Tatar, Y. Toda, T. Watari and M. Yamazaki, “New Aspects of Heterotic–F Theory Duality,” Nucl. Phys. B **806**, 224 (2009) [arXiv:0805.1057 [hep-th]]. H. Hayashi, T. Kawano, R. Tatar and T. Watari, “Codimension-3 Singularities and Yukawa Couplings in F-theory,” Nucl. Phys. B **823**, 47 (2009) [arXiv:0901.4941 [hep-th]].
- [32] P. S. Aspinwall and R. Kallosh, “Fixing all moduli for M-theory on $K3 \times K3$,” JHEP **0510**, 001 (2005) [hep-th/0506014].
- [33] A. P. Braun, Y. Kimura and T. Watari, “The Noether-Lefschetz problem and gauge-group-resolved landscapes: F-theory on $K3 \times K3$ as a test case,” JHEP **1404**, 050 (2014) [arXiv:1401.5908 [hep-th]].
- [34] C. Beasley, J. J. Heckman and C. Vafa, “GUTs and Exceptional Branes in F-theory - II: Experimental Predictions,” JHEP **0901**, 059 (2009) [arXiv:0806.0102 [hep-th]].
- [35] M. Buican, D. Malyshev, D. R. Morrison, H. Verlinde and M. Wijnholt, “D-branes at Singularities, Compactification, and Hypercharge,” JHEP **0701**, 107 (2007) [hep-th/0610007].
- [36] R. Tatar and T. Watari, “GUT Relations from String Theory Compactifications,” Nucl. Phys. B **810**, 316 (2009) [arXiv:0806.0634 [hep-th]].
- [37] T. W. Grimm, T. W. Ha, A. Klemm and D. Klevers, “Computing Brane and Flux Superpotentials in F-theory Compactifications,” JHEP **1004**, 015 (2010) [arXiv:0909.2025 [hep-th]].
- [38] The last reference of [28].
- [39] M. Berg, M. Haack and B. Kors, “On the moduli dependence of nonperturbative superpotentials in brane inflation,” hep-th/0409282. M. Haack, R. Kallosh, A. Krause,

- A. D. Linde, D. Lust and M. Zagermann, “Update of D3/D7-Brane Inflation on $K3 \times T^{**2}/Z(2)$,” Nucl. Phys. B **806**, 103 (2009) [arXiv:0804.3961 [hep-th]].
- [40] H. Ooguri and C. Vafa, “On the Geometry of the String Landscape and the Swampland,” Nucl. Phys. B **766**, 21 (2007) [hep-th/0605264].